# A Flexagon, Flexatube, and Bregdoid Book of Designs

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**Abstract**

Maps and cutouts for a variety of flexagons are presented, emphasizing those which can be cut out, mostly from single sheets of paper. Since TeX may not align front and back images, and in any event if cutting up the booklet is not desired, the .eps files can be printed directly to get sheets suitable for cutting. In the same spirit, only those sheets which are going to be used right away need be printed.

## Contents

# 1 Introduction

Flexagons can become fairly complicated. The ones based on triangles are most conveniently made from long strips of paper; a roll of adding machine or calculator tape is ideal for this purpose given its convenient width. Crooked strips can be gotten by gluing faces together, or just cutting out segments and then joining them together. Leaving one extra triangle in each segment for overlapping and later gluing leads to efficient constructions.

Coloring the triangles is another problem, which can be done with crayons or markers once it is known which colors ought to be used. Aside from copying an already existent design, this is best done by drawing the Tukey triangles and then lettering or numbering the triangles in the strip. That information is sufficient to fold up the strip, since pairs of consecutive numbers are to be hidden by folding them together. Painting can be done before folding by following a color code for the numbers, or after the folding is done, when the faces can be painted wholesale, or even embellished with designs.

Other flexagons, even the ones folded from “straight” strips, require a higher degree of preparation, although it is relatively easy to assemble a collection of primitive components which later can be glued together according to the necessities of the individual flexagon.

# 2 First Level Tetraflexagon

## 2.1 Tukey triangles

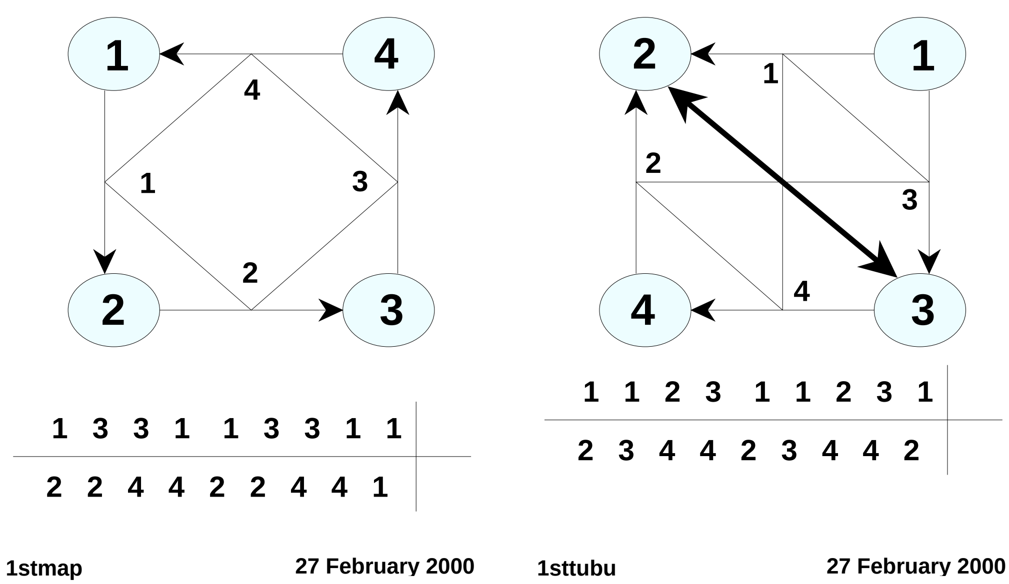



Figure 1: Left: Tukey triangle map for the normal first level tetraflexagon, which consists of a single cycle of four colors. Right: Triangle map for the tubulating first level tetraflexagon, likewise displaying four colors, but which contains a single tubulation.

### 2.2 Flexagon permutations

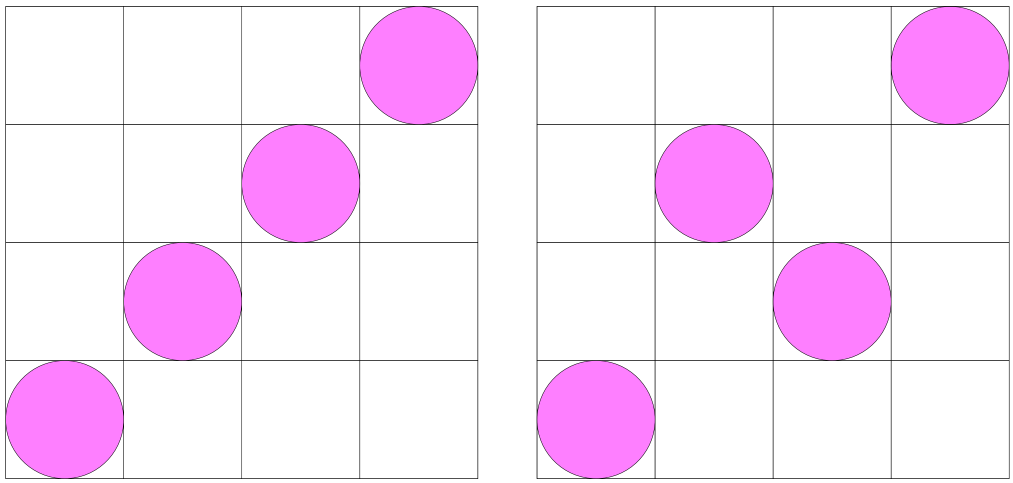


Figure 2: Left: Permutation of the squares along the strip for a normal tetraflexagon. They run in order, subject to being flipped over, so the permutation is the identity. Right: Permutation for the tubulating flexagon. The normal order is not preserved, the permutation is not the identity.

### 2.3 First level normal tetraflexagon cutout

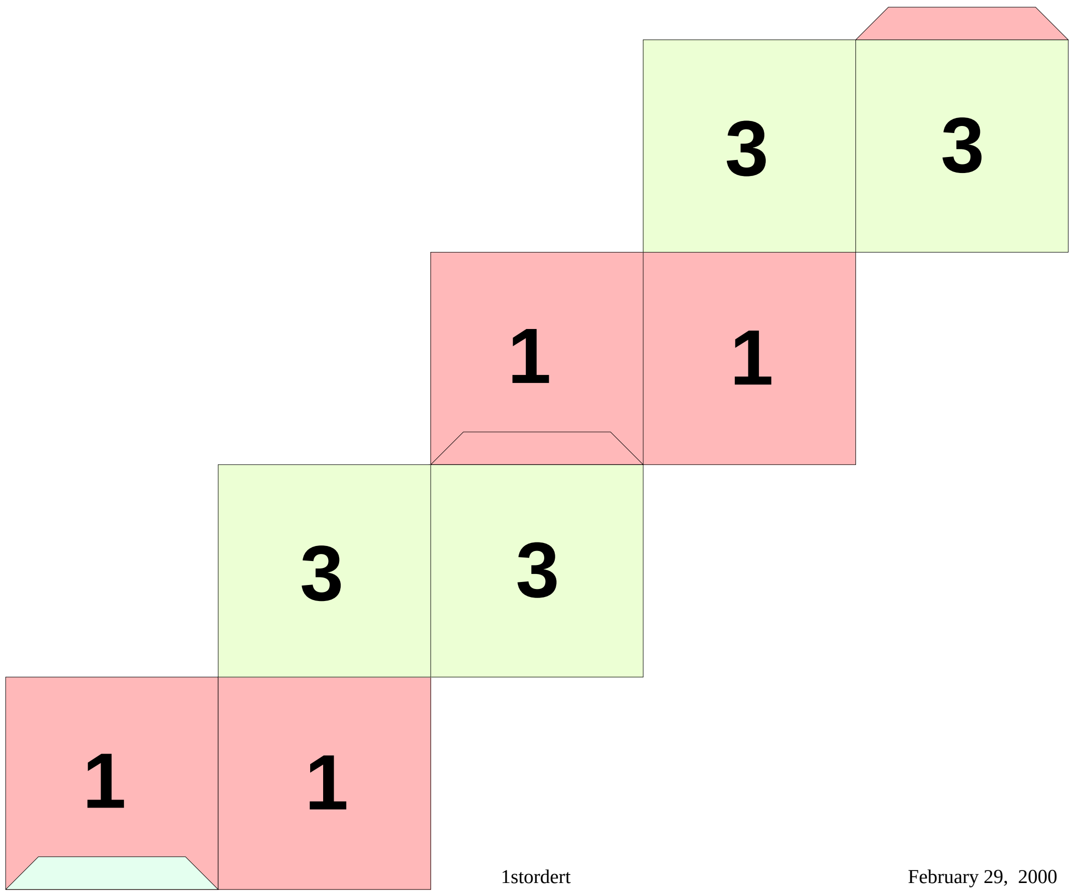



Figure 3: Top side of the first level normal tetraflexagon cutout. Together with its backside, the figure contains one single tetraflexagon.

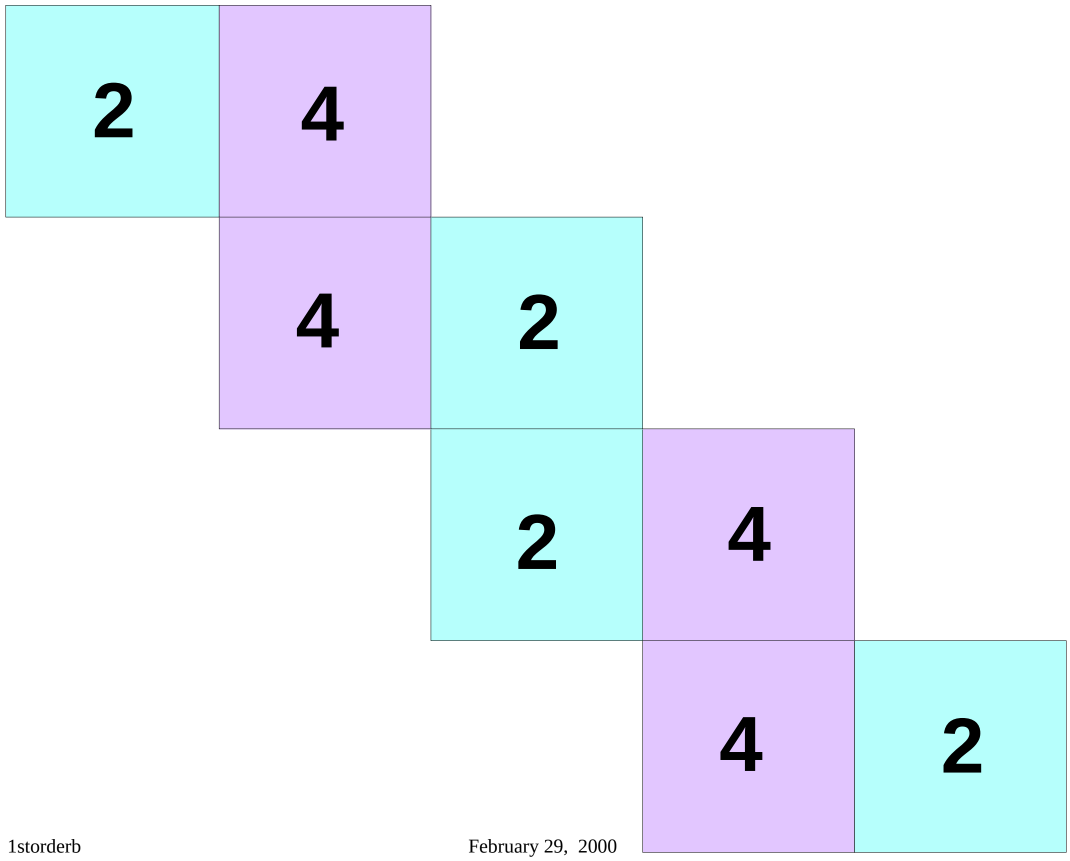



Figure 4: Bottom side of the first level normal tetraflexagon cutout.

## 2.4 First level tubulating tetraflexagon cutout

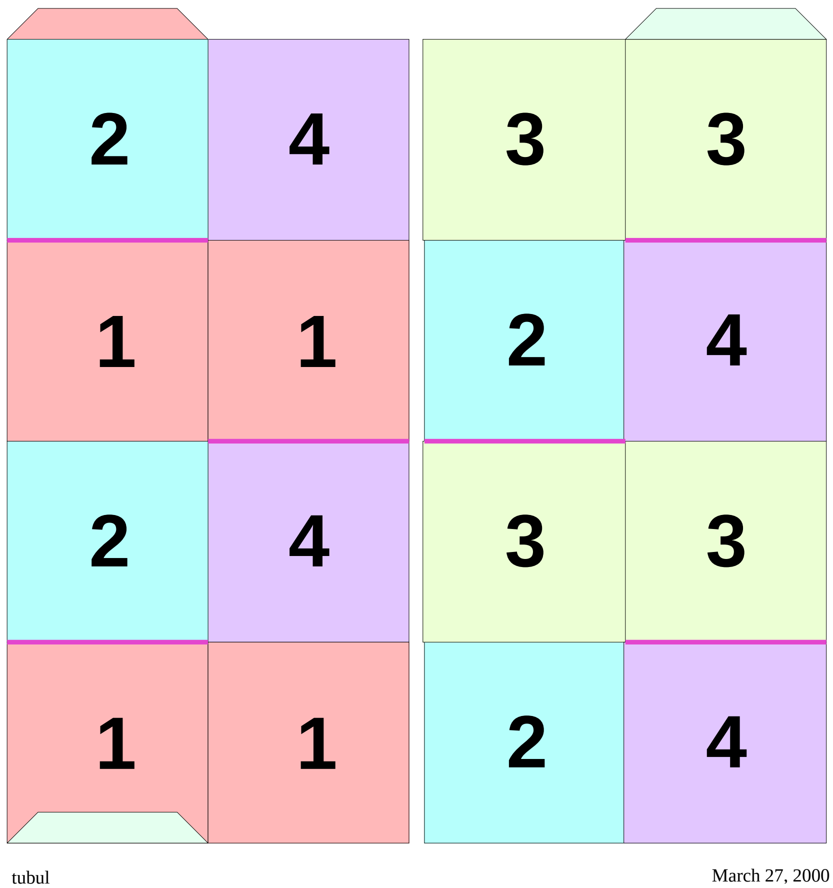



Figure 5: Top side of the first level tubulating tetraflexagon cutout, characterized by the sign sequence (+ + - -). The cutout is small enough that both top and bottom for two sectors are shown, enough to make two separate flexagons.

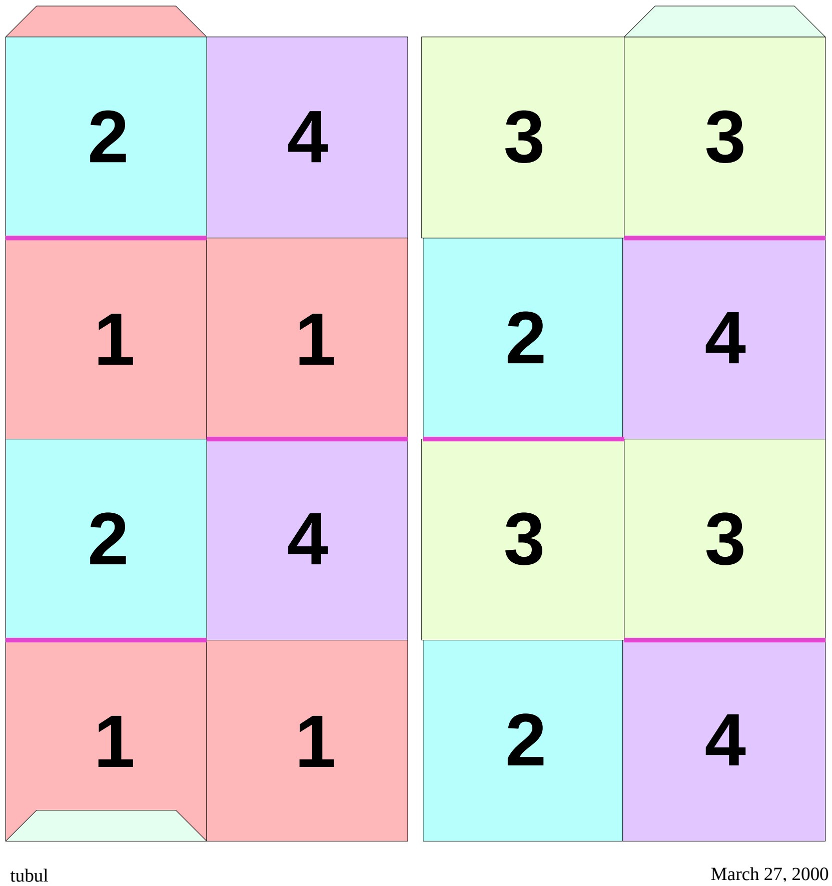



Figure 6: reverse side of the first level tubulating tetraflexagon cutout.

## 2.5 Alternative first level tubulating tetraflexagon cutout

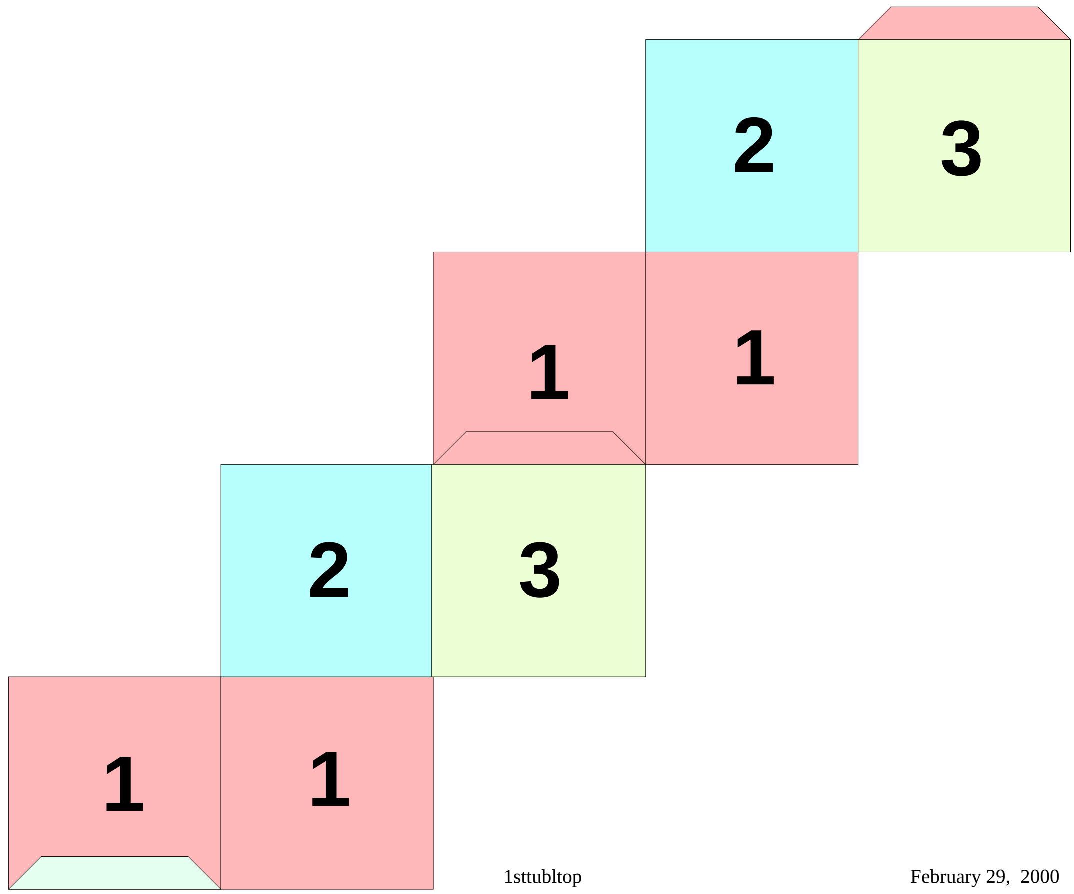



Figure 7: Top side of the alternative first level tubulating tetraflexagon cutout, characterized by the sign sequence $(++++)$. The figure, taken with its backside, makes one single flexagon consisting of two sectors.

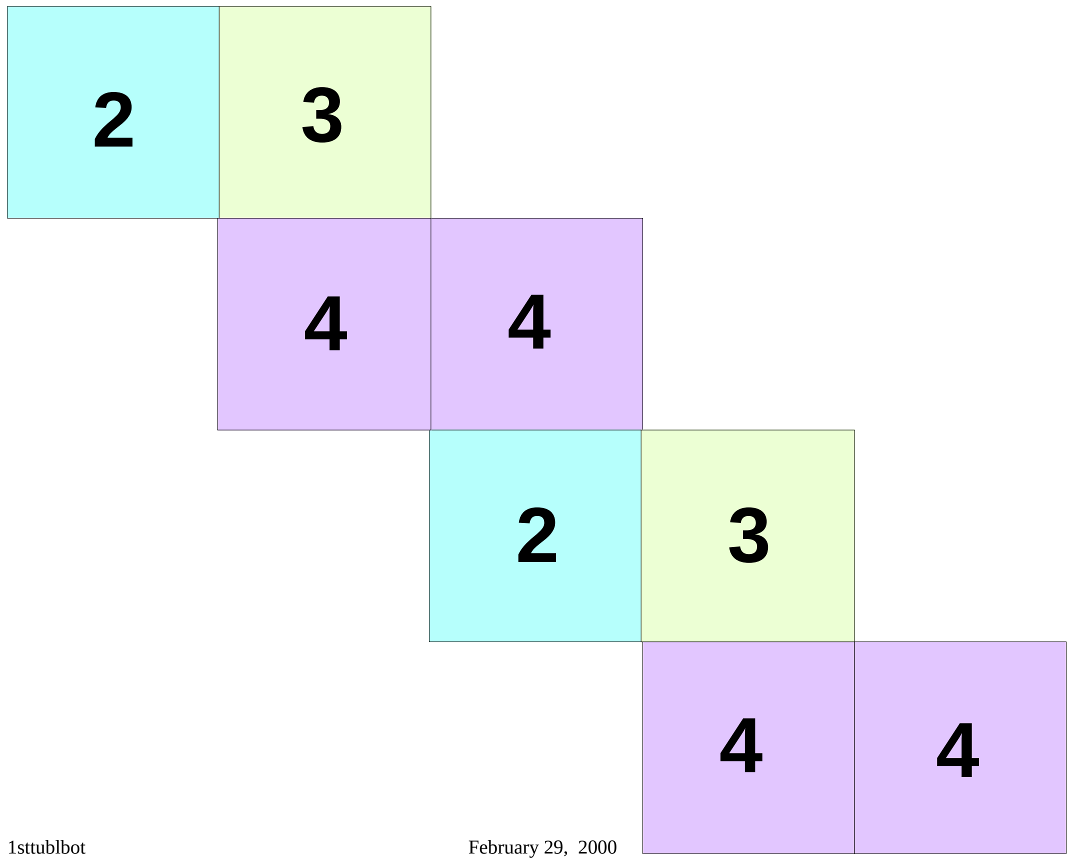



Figure 8: Bottom side of the alternative first level tubulating tetraflexagon cutout.

## 3 Binary Tetraflexagon

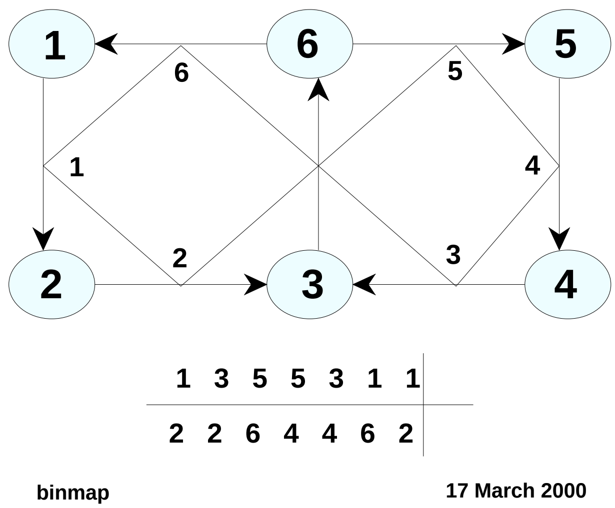



Figure 9: The binary tetraflexagon has two cycles, each of which has two vertices in common with the other.

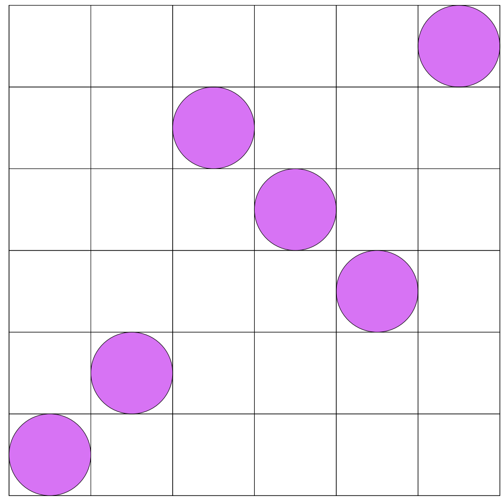


Figure 10: Permutation of the squares along the strip for a binary tetraflexagon.

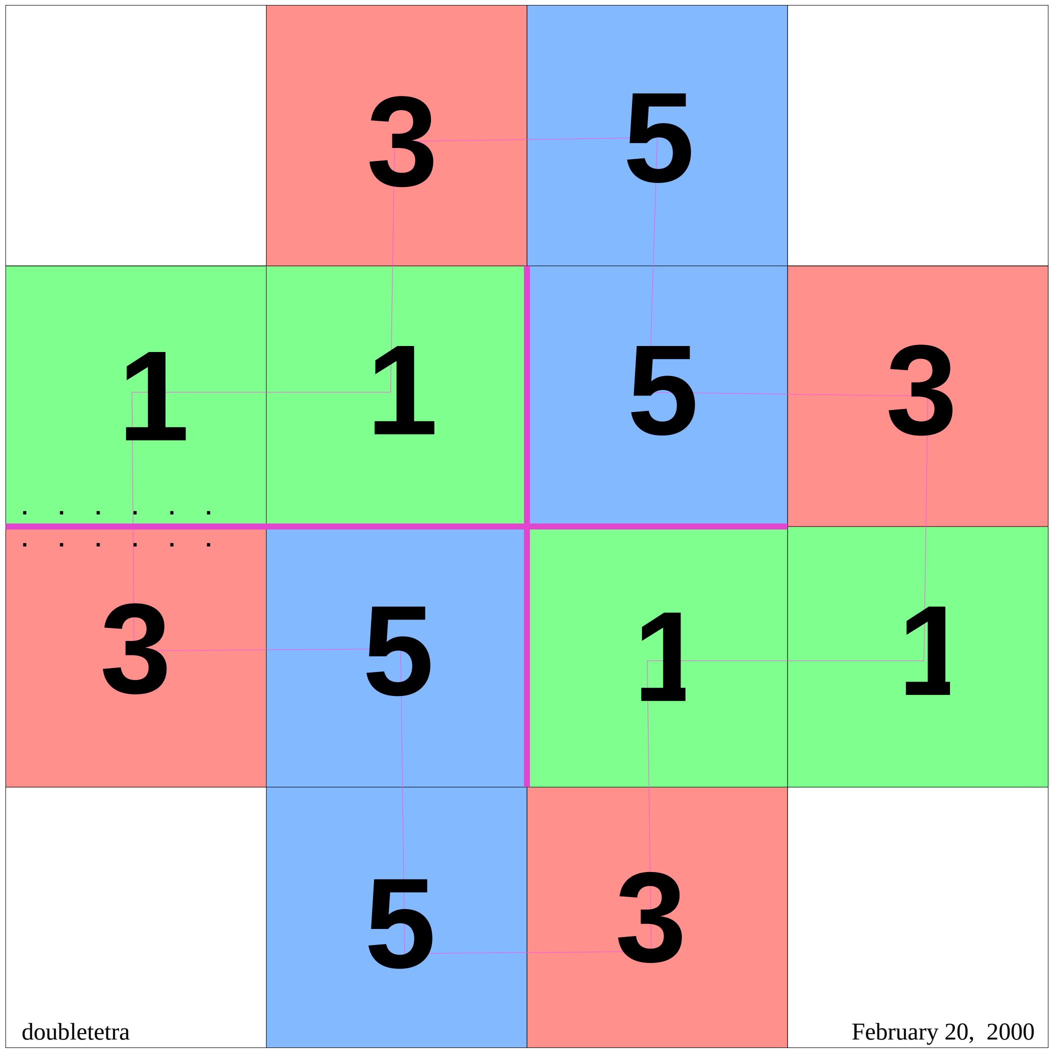



Figure 11: Top side of the first level tetraflexagon cutout. The flexagon has six faces, so this cutout provides material for both of the sectors needed for the flexagon. Cuts should be made along the heavy lines, edges joined (using an extra little strip of paper taken from an unused spot) along the edges marked by dots. Together with the backside, the figure makes one individual flexagon.

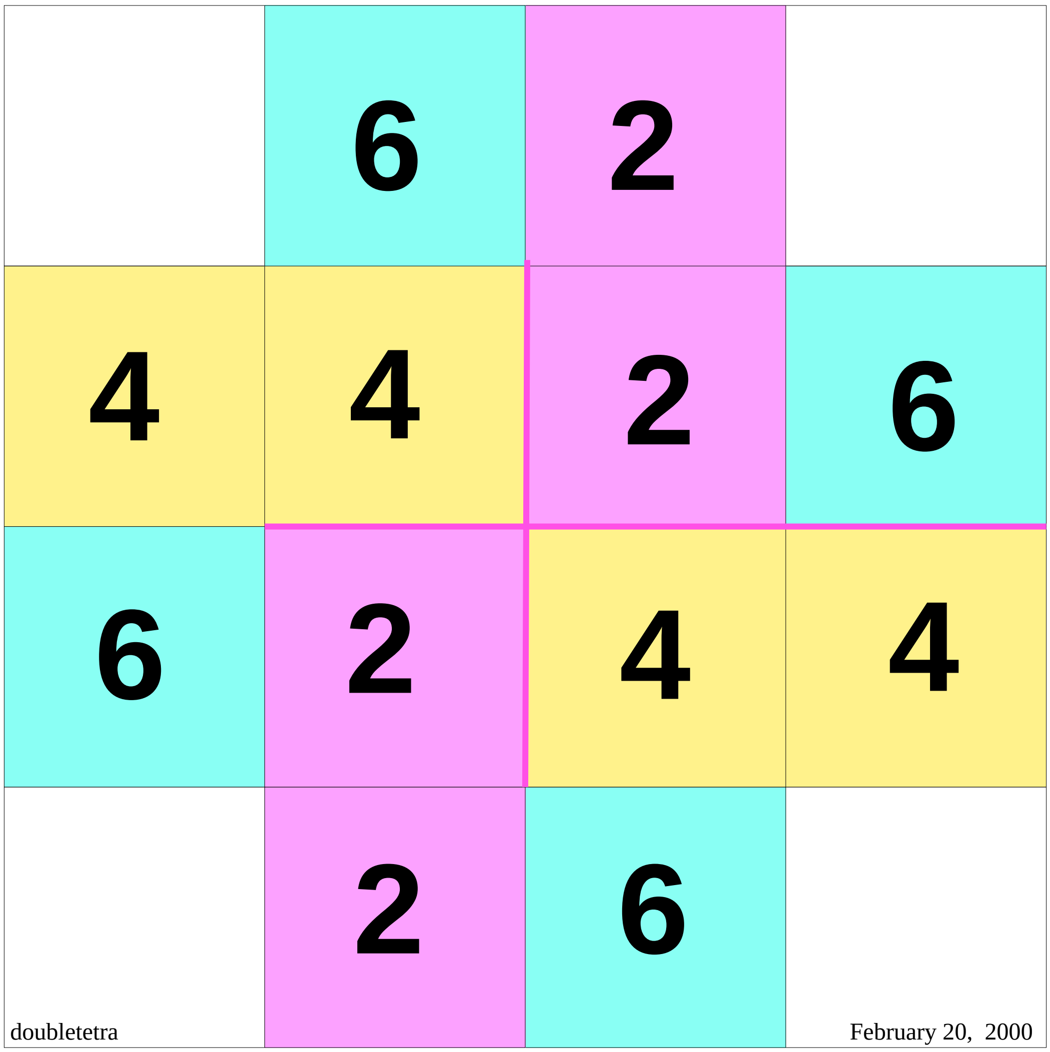



Figure 12: Bottom side of the first level binary tetraflexagon cutout.

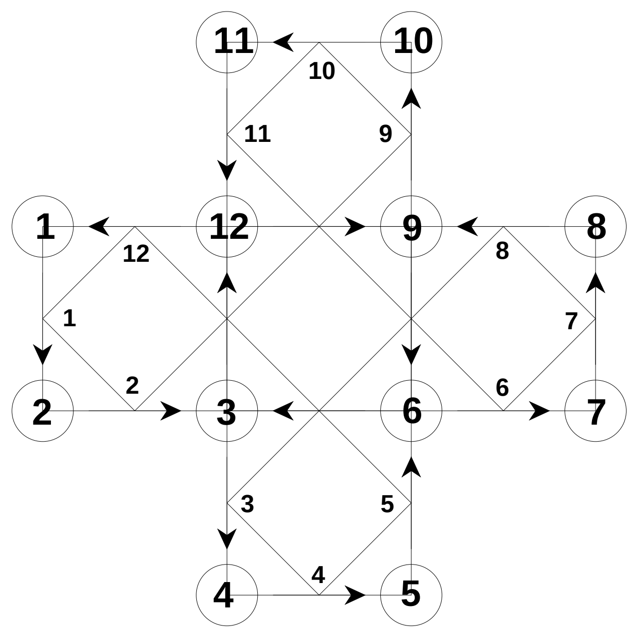
# 4 Second Level Tetraflexagon

Starting from the generic second level tetraflexagon map, which consists of a node linked to four surrounding nodes, any of the linked nodes can be dropped for a total of sixteen possibilities, not all of which are symmetrically independent. Thus one cound drop one node, an opposite or an adjacent pait, or three of the four. That makes six combinations, plus the full second order flexagon or just the full first order flexagon.

Amongst all of these, various possibilities for tubulations exist.

## 4.1 Second level normal tetraflexagon

The generic second level tetraflexagon has twelve faces, and is still quite easy to fold up from a previously prepared strip of paper.




| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 9 | 11 | 11 | 7 | 7 | 9 | 3 | 5 | 5 | 1 | 1 |
| 2 | 2 | 10 | 10 | 12 | 6 | 8 | 8 | 4 | 4 | 6 | 12 | 2 |

Figure 13: Since each edge of the first level tetraflexagon spawns two new vertices, the full second level tetraflexagon has 12 vertices.

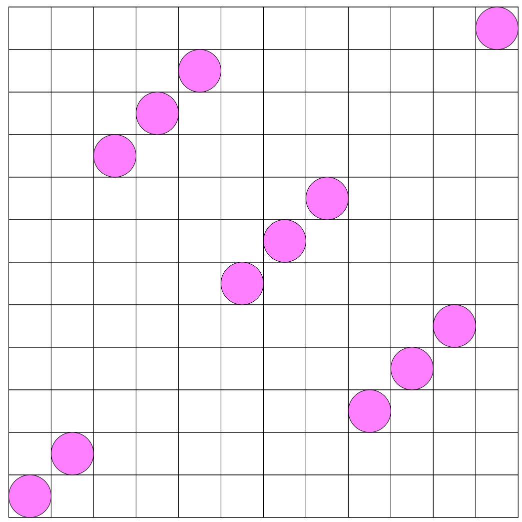


Figure 14: Permutation of the squares along the strip for a normal second level tetraflexagon. They run in order, subject to packages being flipped over.

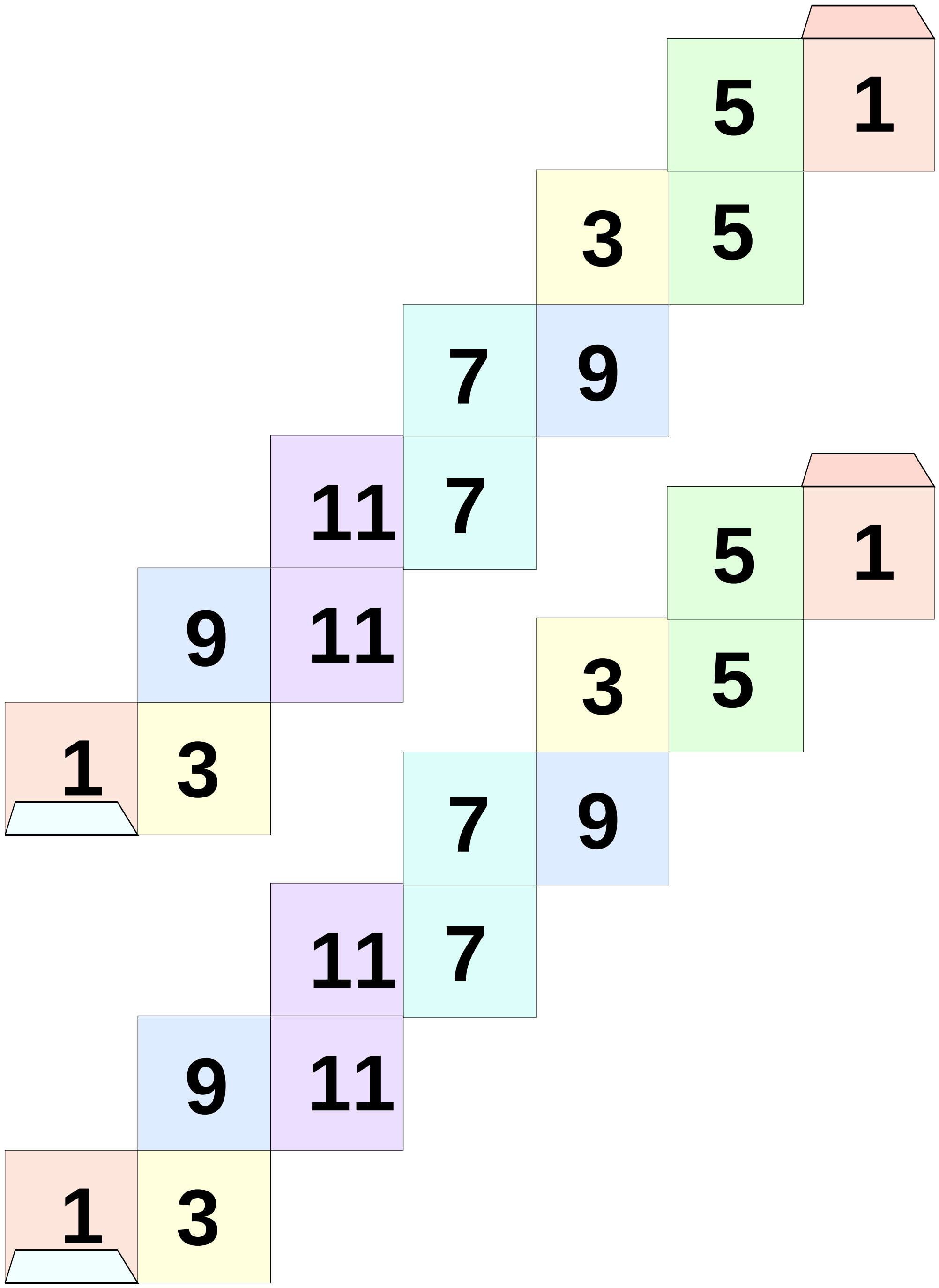



Figure 15: Strips from which the second level tetraflexagon can be folded. One strip folds into a single sector, so two are needed altogether.

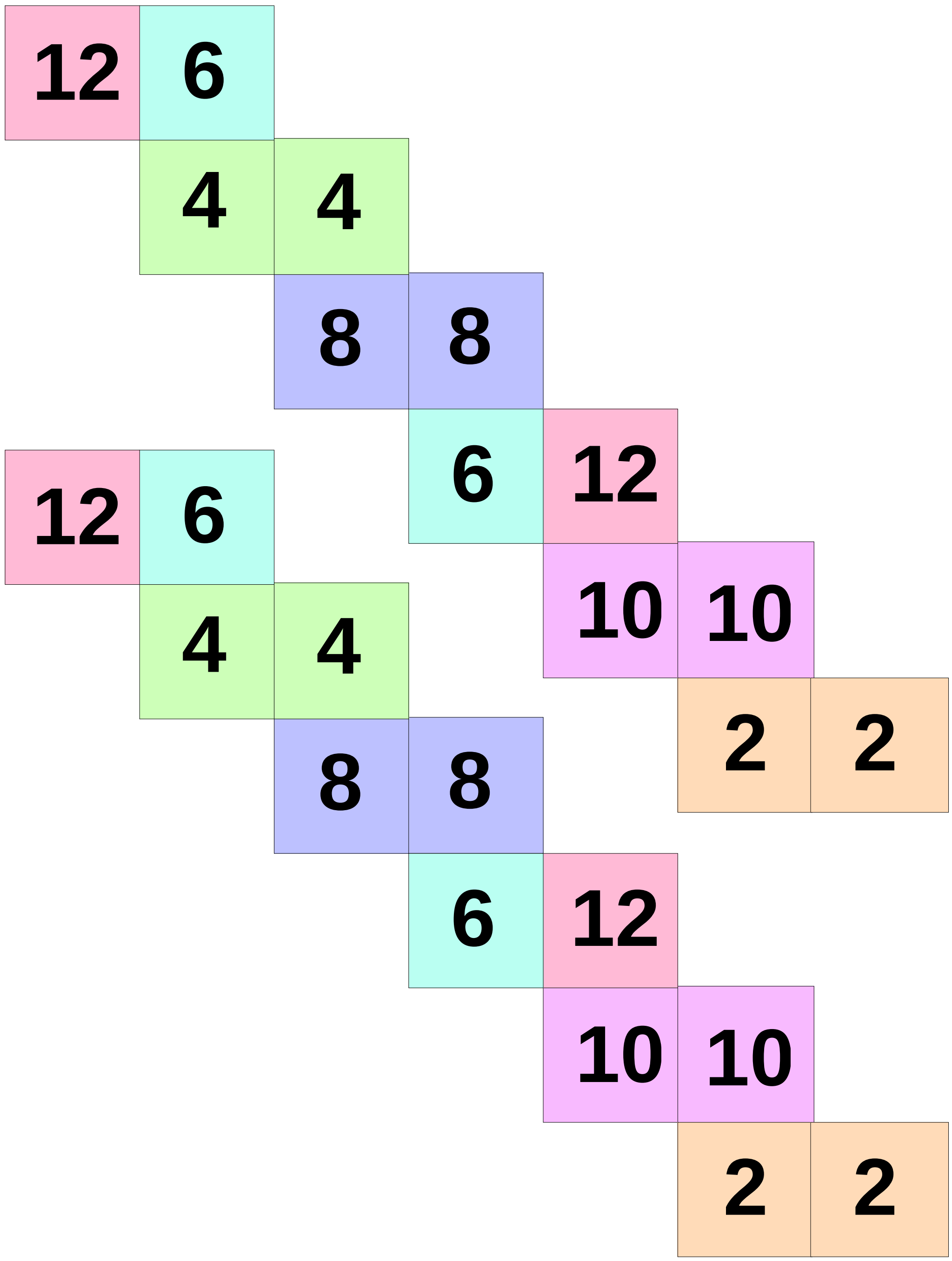



Figure 16: Back side of the two strips for the second level normal tetraflexagon. One sector per strip.

## 4.2 Second level tubulating tetraflexagon

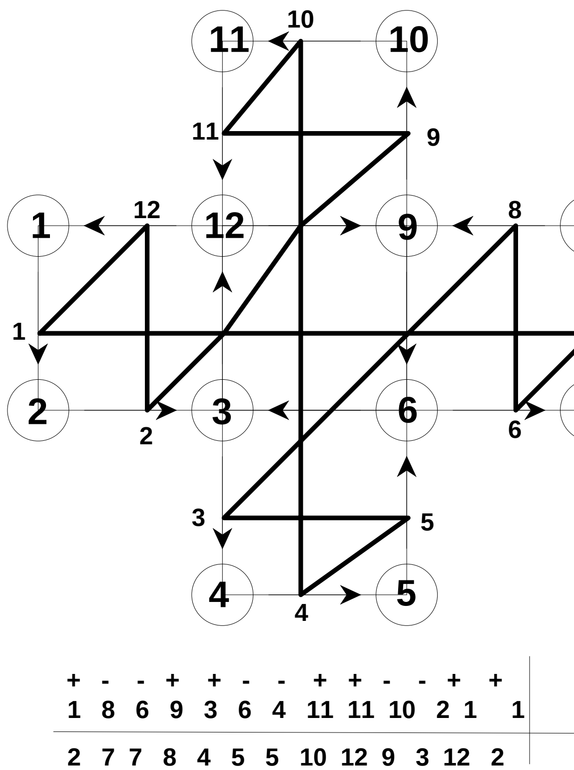



Figure 17: The 12 vertices of the generic second level tetraflexagon can be connected to produce tubulations.

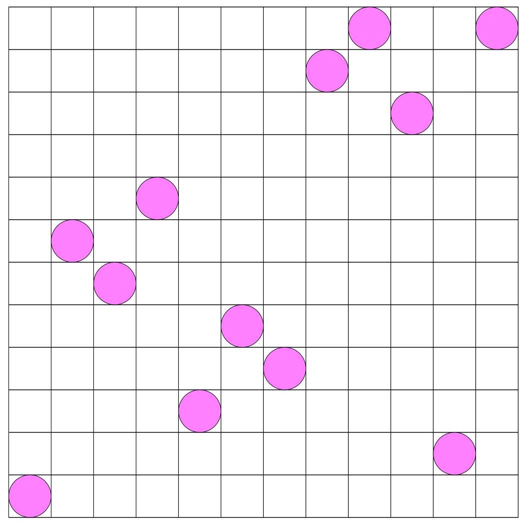


Figure 18: Permutation of the squares along the strip for a tubulating second level tetraflexagon.

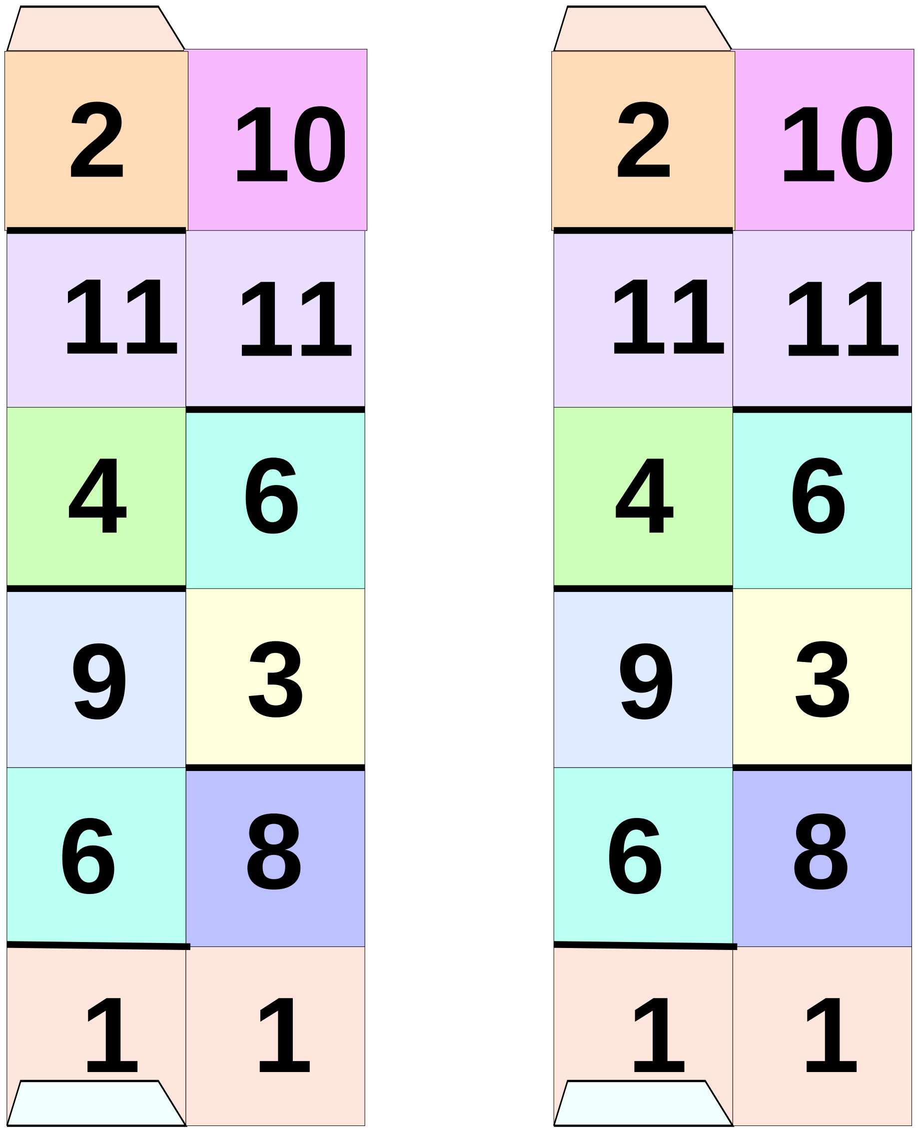



Figure 19: Top side of the tubulating second level tetraflexagon. Two strips are shown, sufficient to make both of the two sectors which the flexagon requires.

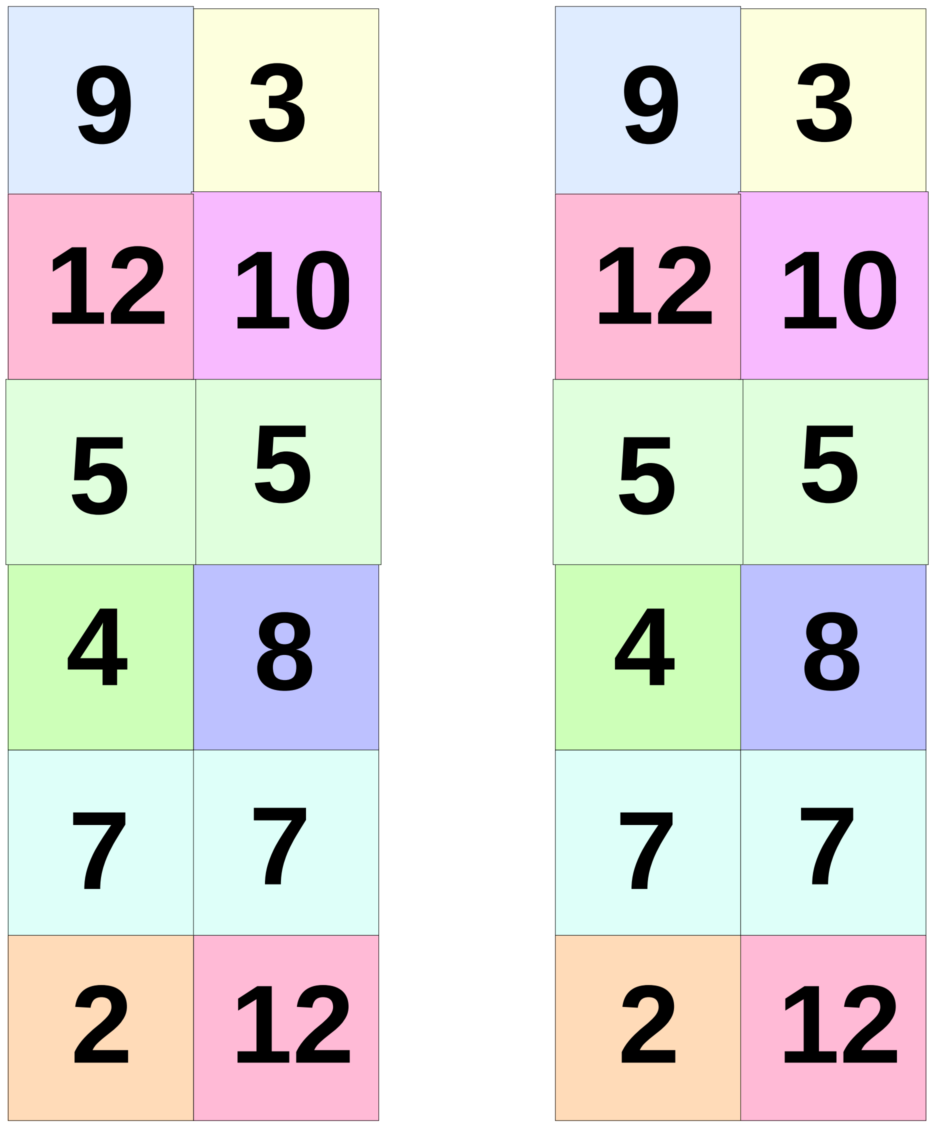



Figure 20: Bottom side of the tubulating second level tetraflexagon. Two strips are shown, sufficient to make both of the two sectors which the flexagon requires.

# 5 Third level tetraflexagon

The generic third level tetraflexagon has thirty six faces, and is still easy to fold up from a previously prepared strip of paper. However, rather than many pages of explicitly numbered squares, the templates described in the next section should be consulted.

The easiest way to create a generic flexagon is to prepare some uncolored and unnumbered segments corresponding to the sign sequence $(+ + + + \cdots)$ with tabs on the end. They can later be joined and labelled as required. It is especially easy with triflexagons because a long straight strip of paper only has to be marked off into triangles. Polygons with more sides still use an essentially straight strip of paper, but with little meanders which have to be incorporated since the beginning.

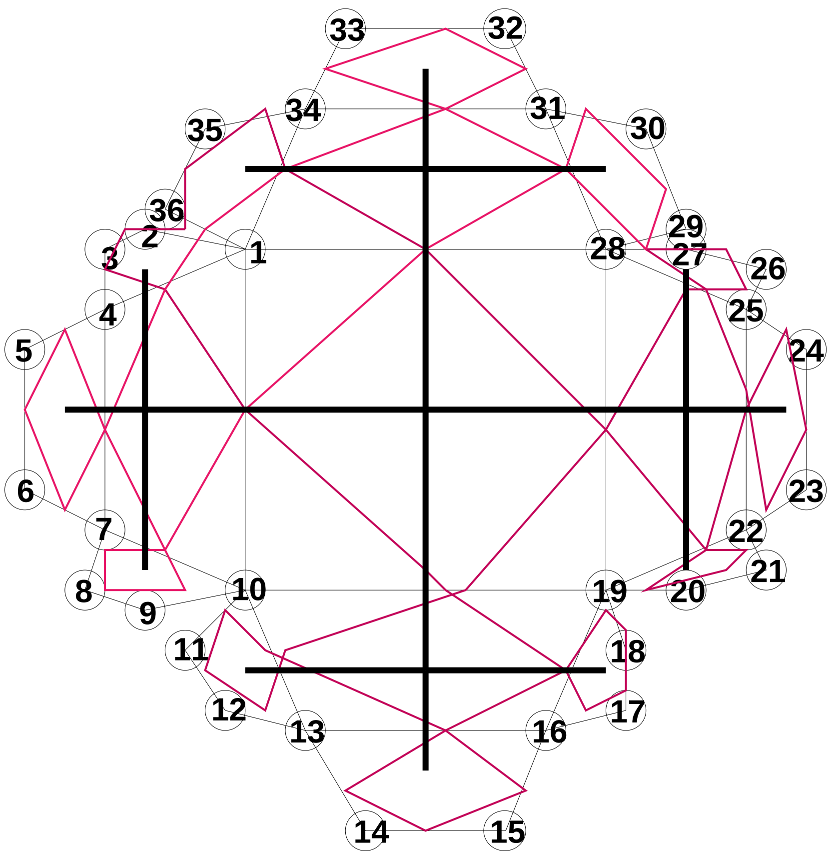



Figure 21: Since each of the twelve edges of the second level tetraflexagon spawns two new vertices, the full second level tetraflexagon has 36.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 1 | 3 | 3 | 17 | 17 | 19 | 13 | 15 | 15 | 11 | 11 | 13 | 25 | 27 | 27 | 23 | 23 | 25 | 19 | 21 | 21 | 35 | 35 | 37 | 31 | 33 | 33 | 29 | 29 | 31 | 7 | 9 | 9 | 5 | 5 | 7 | 1 |
| 2 | 2 | 4 | 16 | 18 | 18 | 14 | 14 | 16 | 10 | 12 | 12 | 26 | 26 | 28 | 22 | 24 | 24 | 20 | 20 | 22 | 34 | 36 | 36 | 32 | 32 | 34 | 28 | 30 | 30 | 8 | 8 | 10 | 4 | 6 | 6 | 2 |

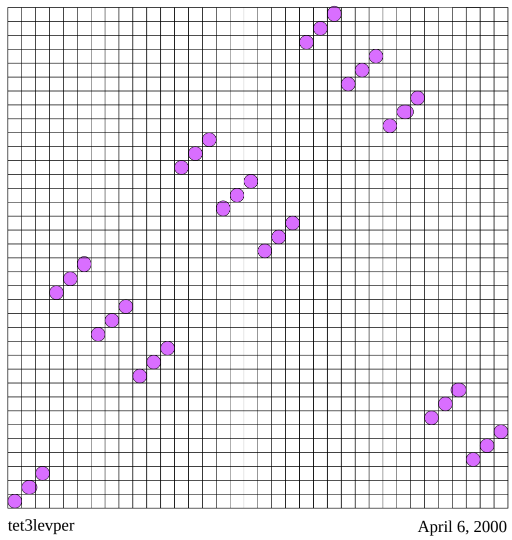



Figure 22: Permutation of the squares along the strip for a third level tetraflexagon.

## 6 General Tetraflexagon, Flexatube, or Bregdoid

To build the more complicated tetraflexagons, the individual squares shown in figures 27 (top side) and 28 (bottom side) can be cut out, labelled, colored, and joined together one by one. Figures 29 and 30 show larger squares which are already joined in groups of four and so would require less cutting and pasting. In either case, the figures should be regarded as mere templates, to be copied in quantities sufficient for any given construction project.

Either flexatubes or bregdoids can be fabricated from the strips shown in Figures 23 (top side) and 24 (bottom side). The figures contain three strips, which should be cut apart. Only one strip is required for a flexatube, so the material is sufficient to make three of them.

The simplest bregdoids are made from strips eight squares long; following which two of the longer strips must be at hand because they are going to be braided together before folding up in flexatube fashion. Therefore the material shown in the figures is only sufficient for three quarters of a bregdoid, and more copies would have to be made before proceeding.

More complex bregdoids can be assembled from longer braids.

Figures 27 and 28 contain the tops and bottoms of some generic squares with tabs for joining them together. They can be used in sufficient numbers to construct the more tortuous tetraflexagons, or even colored with original designs for making flexatubes or bregdoids.

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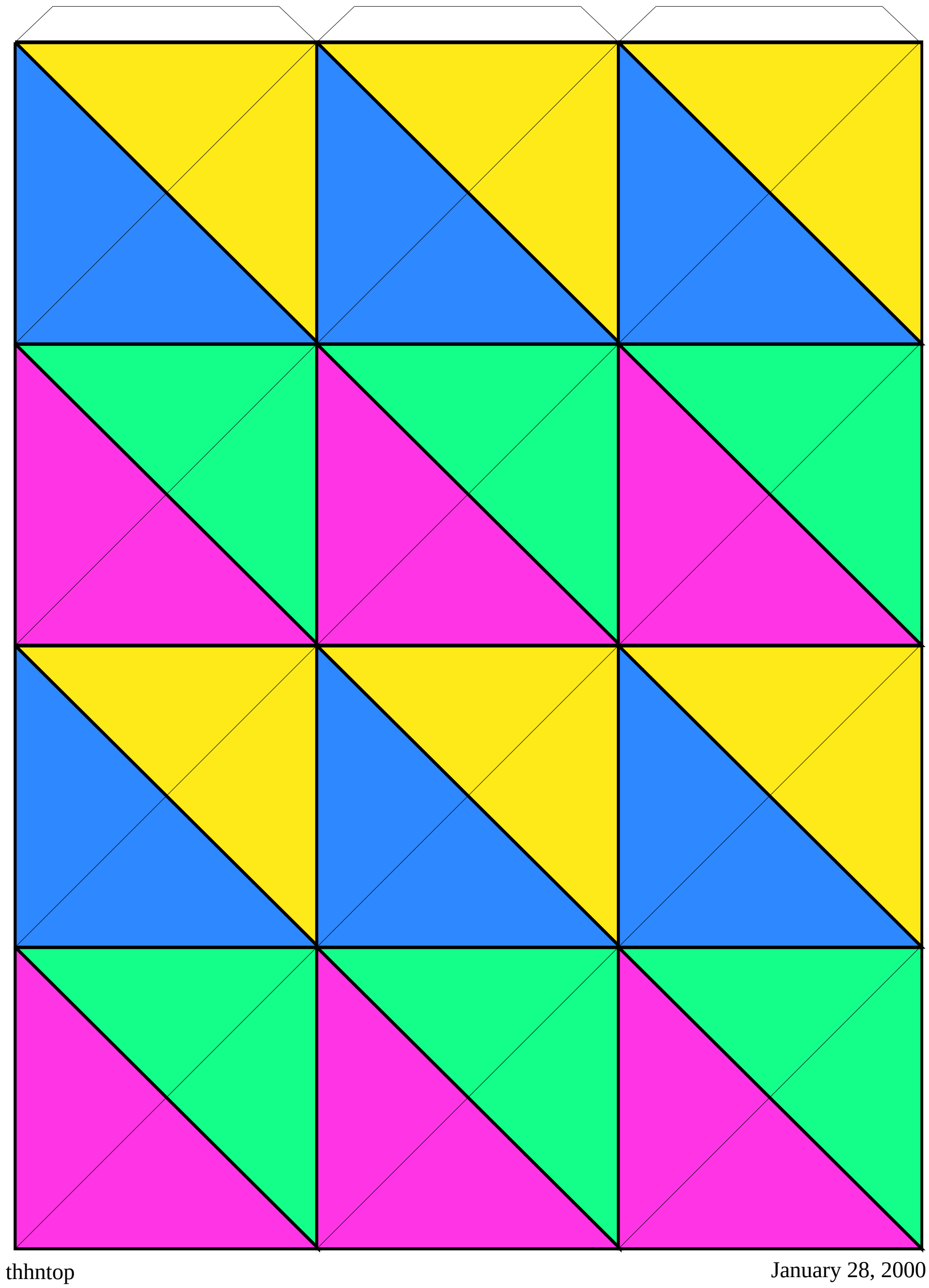



Figure 23: Top side of a flexatube.

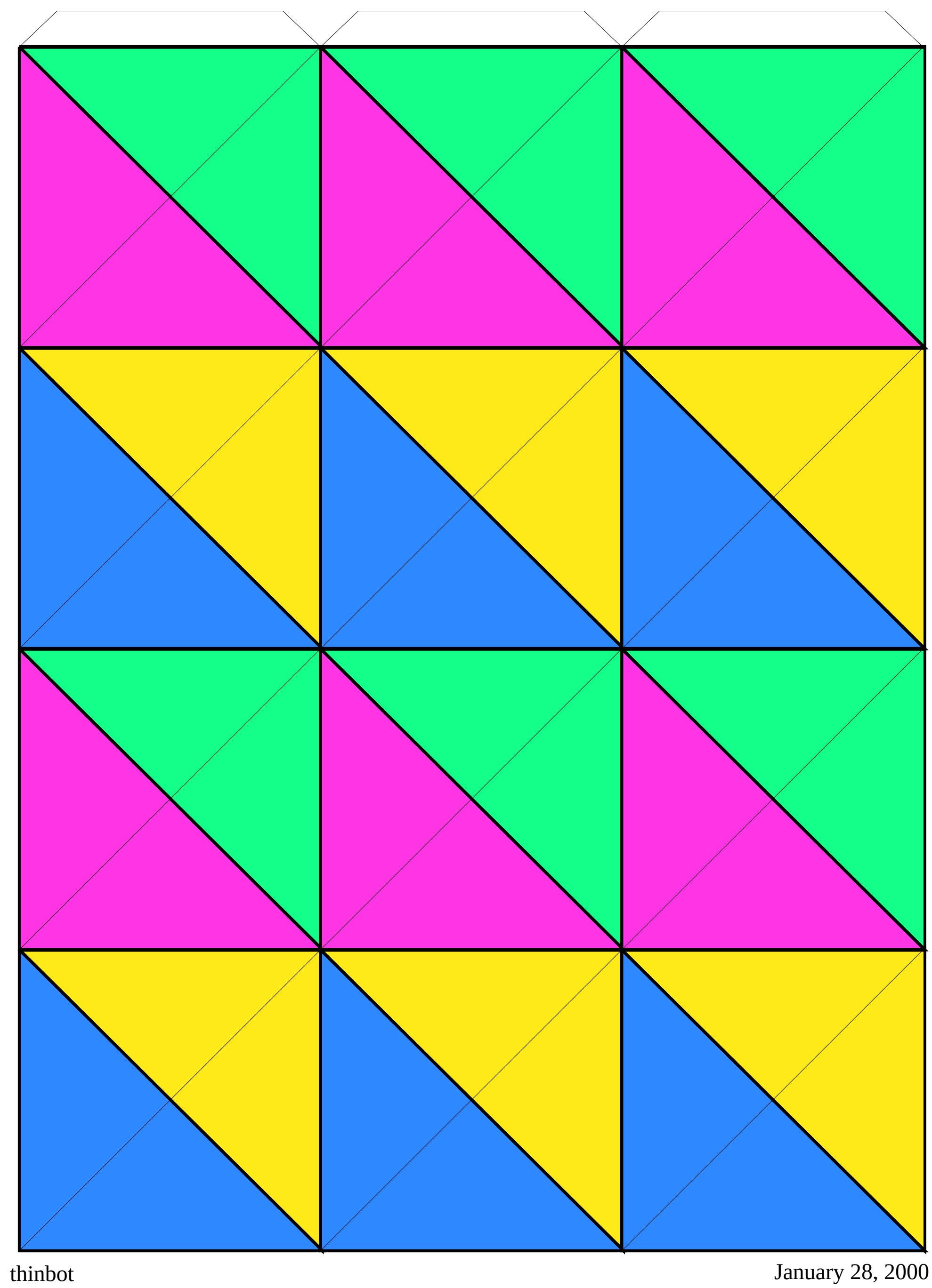



Figure 24: Bottom side of a flexatube.

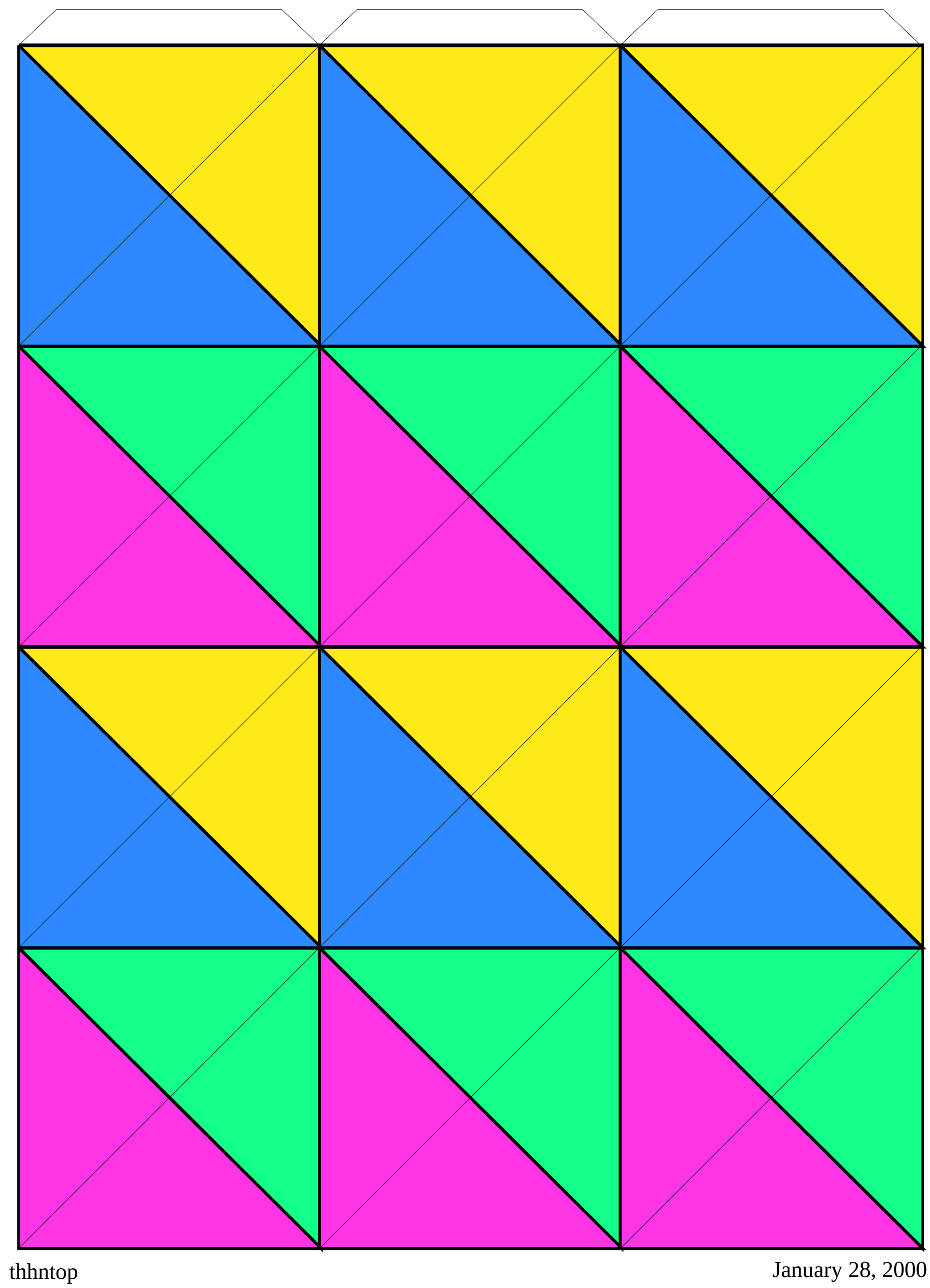



Figure 25: Second copy of the top side of a flexatube, which provides enough additional material to complete the second strand of a bregdoid, and even to make one with slightly longer strands. Otherwise it provides for three more flexatubes.

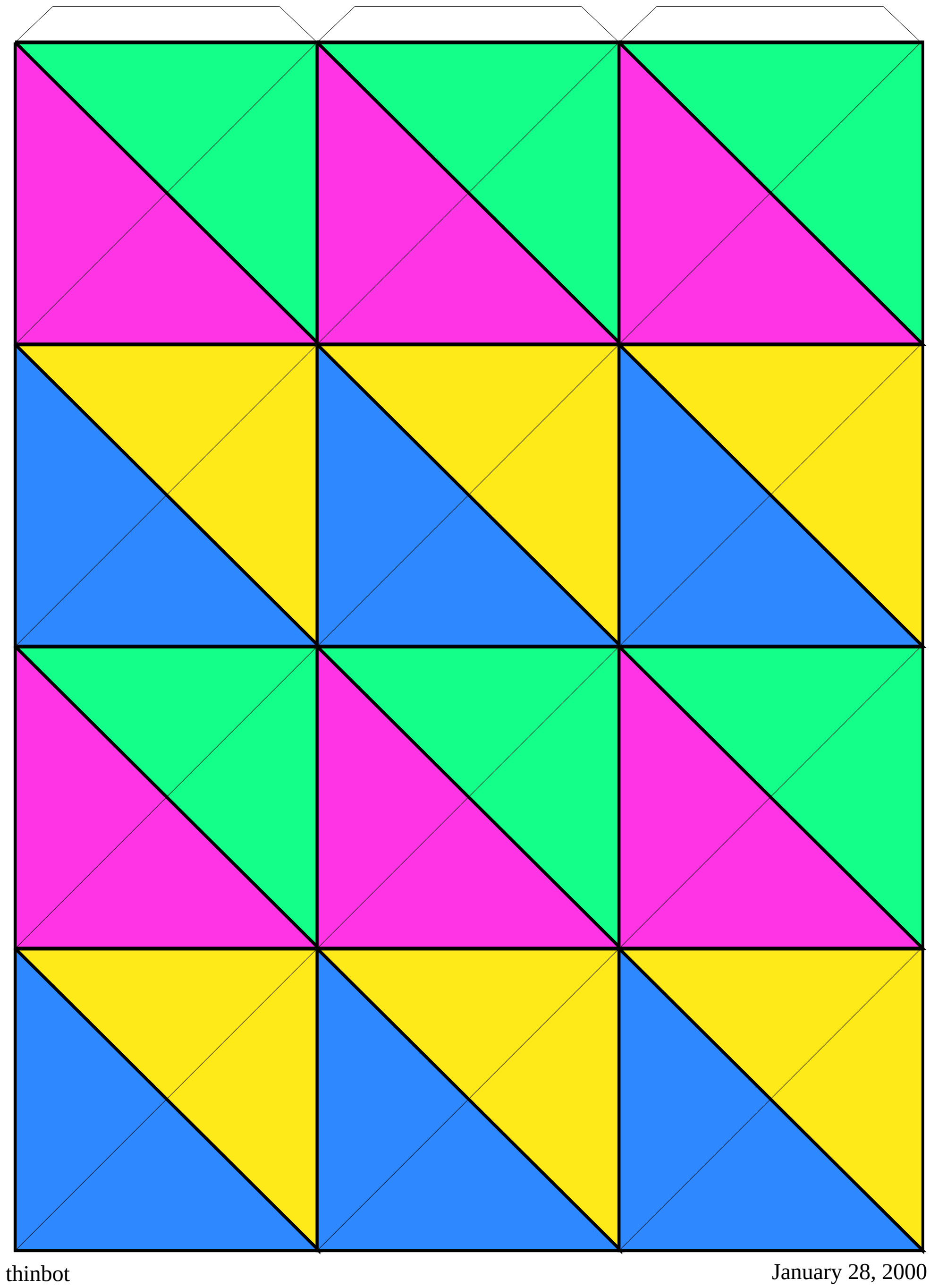



Figure 26: Second copy of the bottom side of a flexatube.

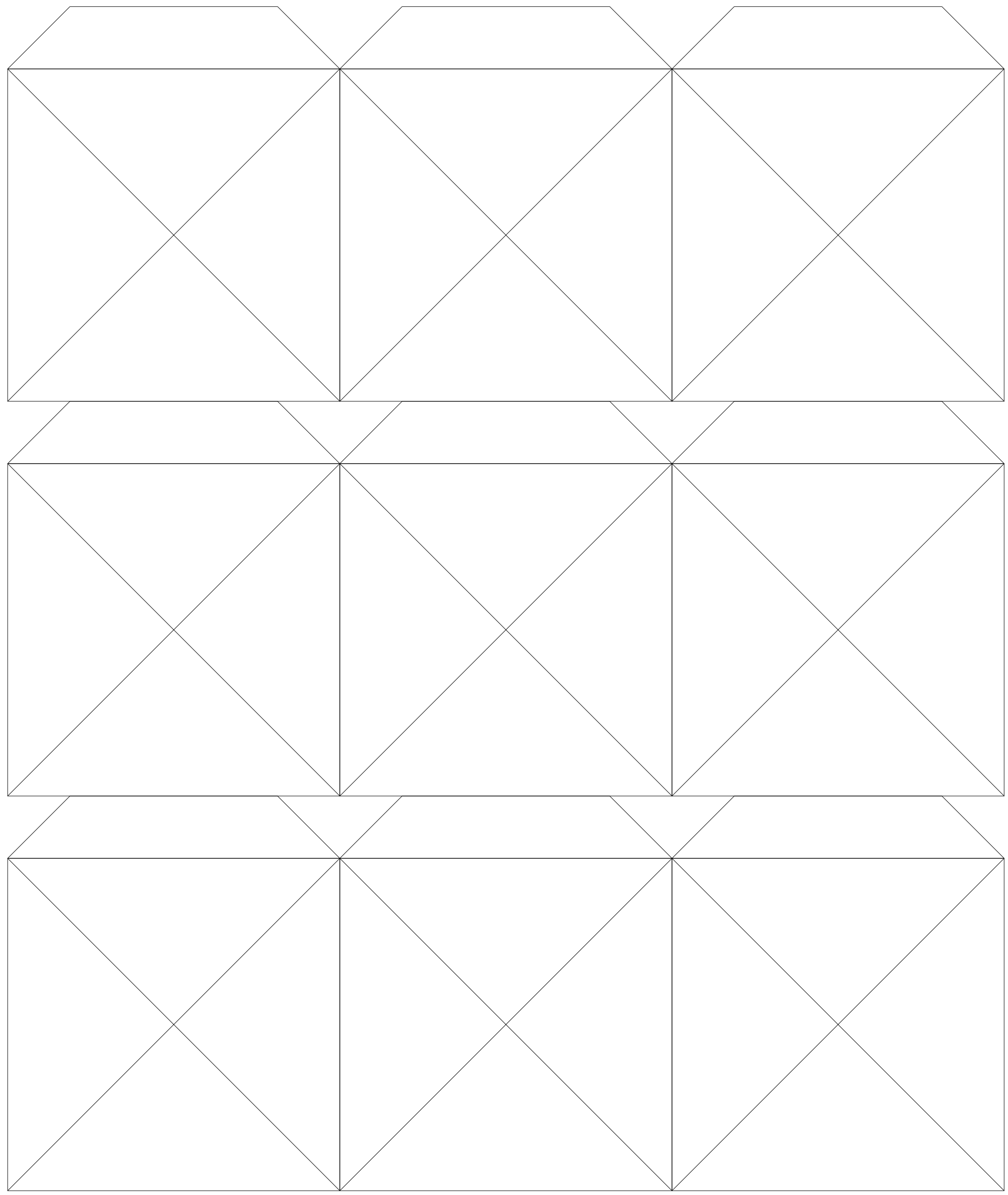


Figure 27: Since even the straight tetraflexagon strips are jagged, the best way to make up a tetraflexagon is to paste individual square cells together. They can be colored and numbered beforehand if that information has already been worked out.

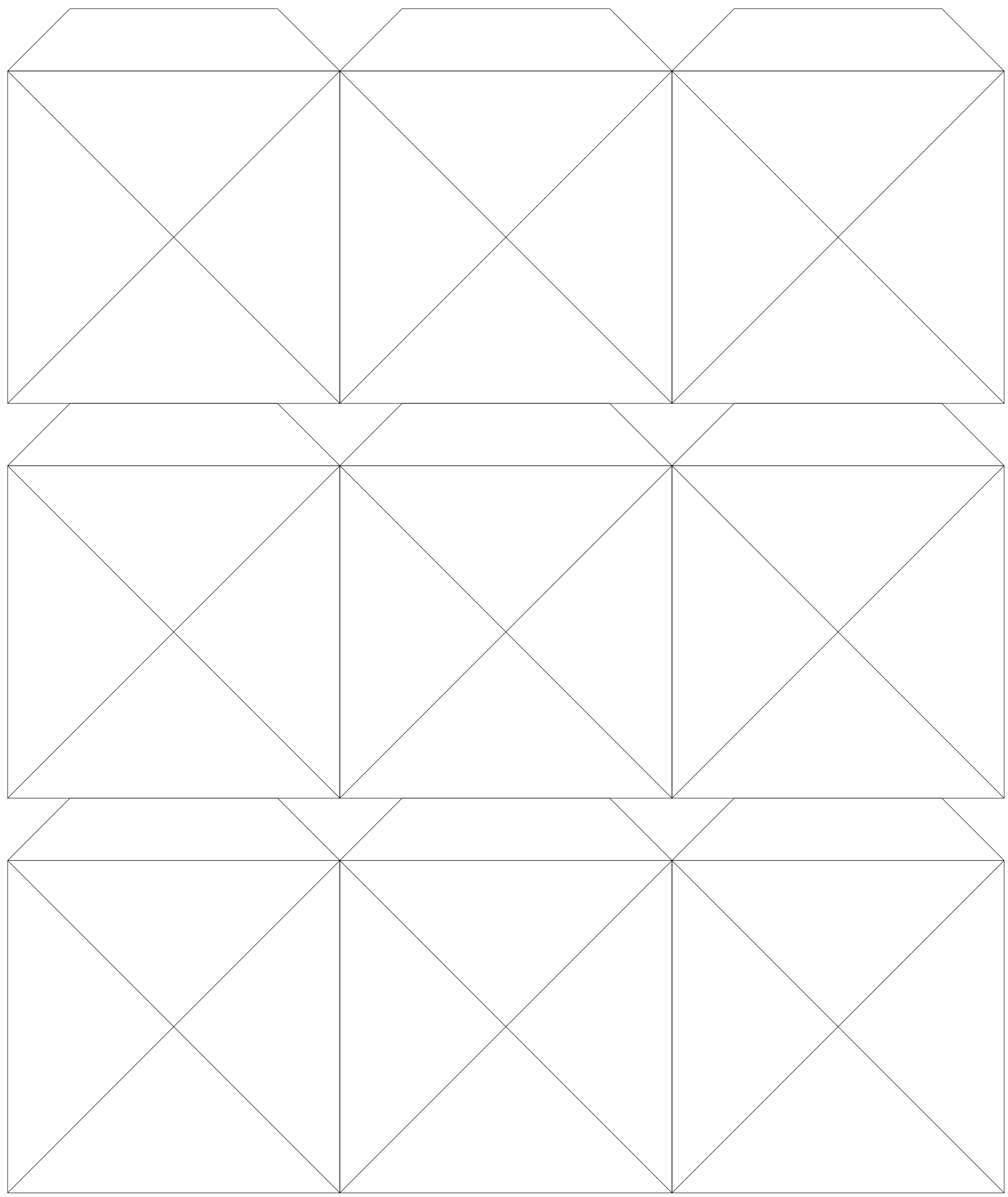


Figure 28: Back side of a panel of square cells.

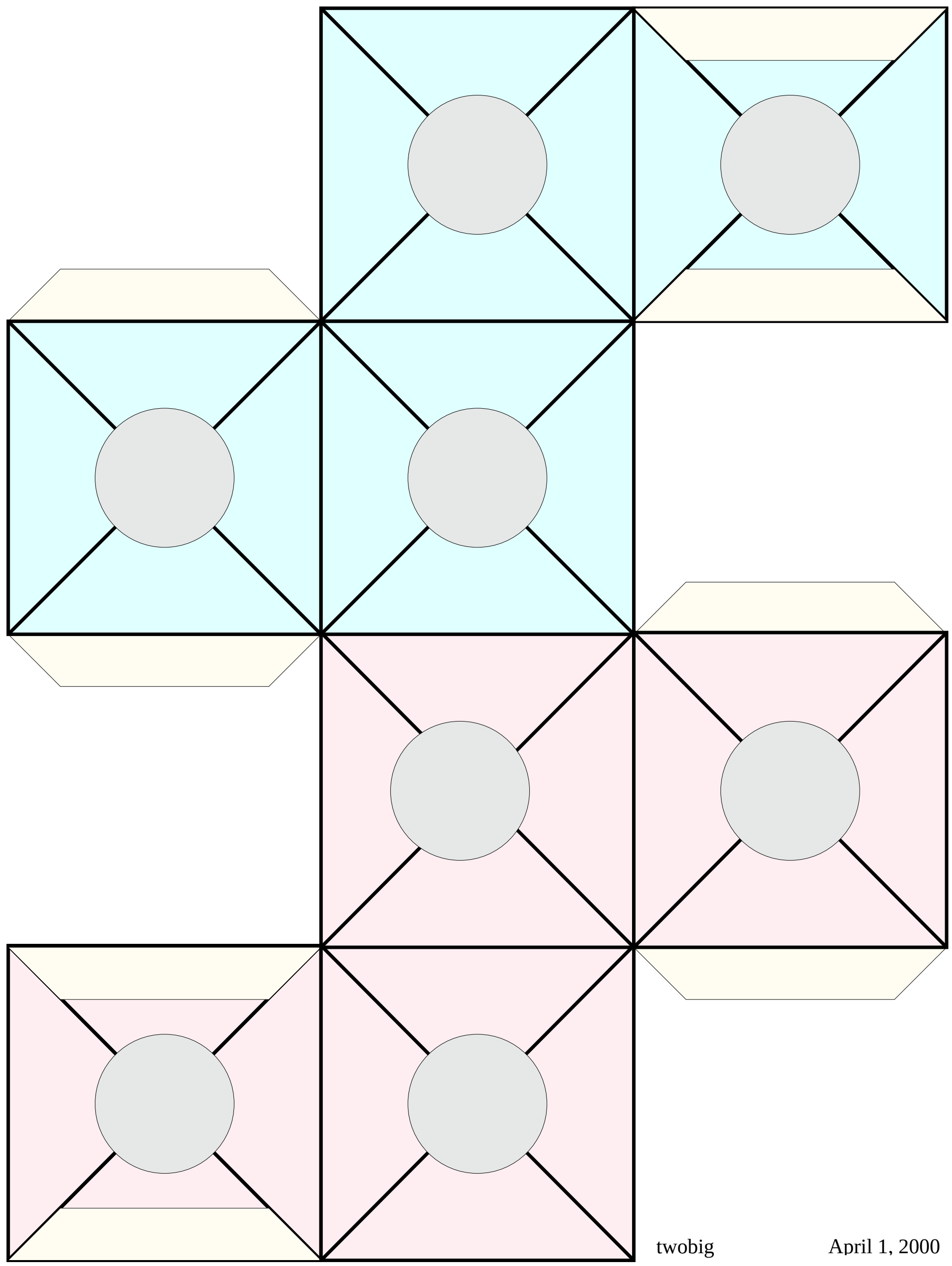



Figure 29: Larger generic squares - top side.

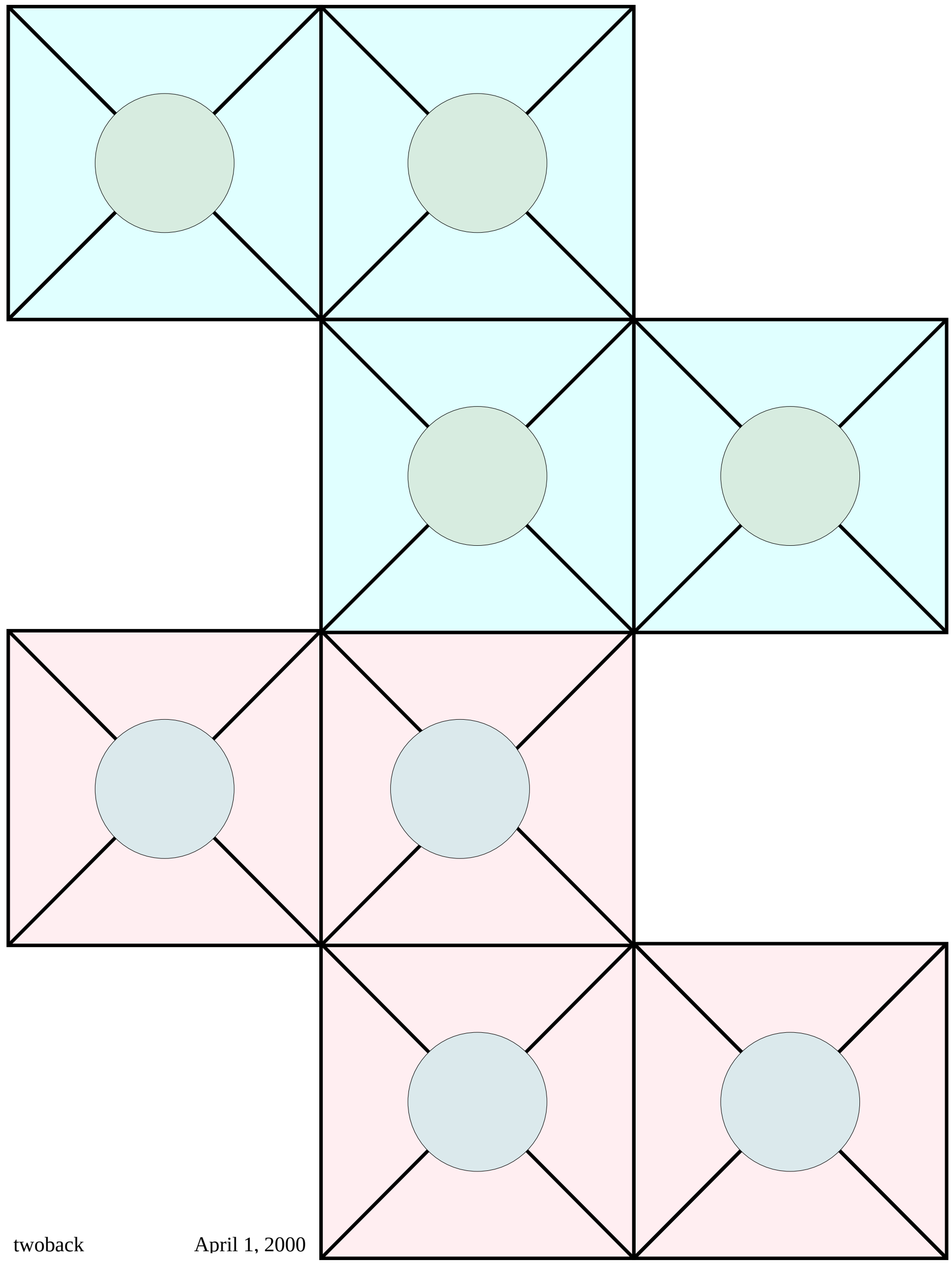
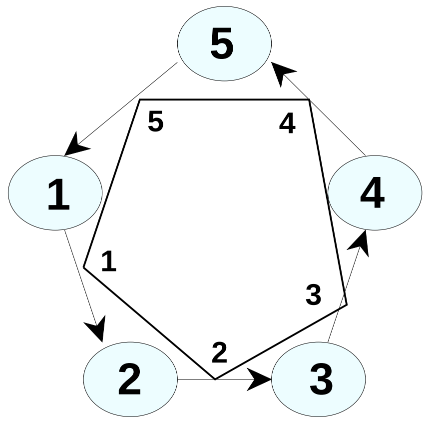



Figure 30: Back side of a panel of larger square cells.

# 7 First Level Flentagon




| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | 5 | 5 | 2 | 2 | 4 | 4 | 1 | 1 |
| 2 | 2 | 4 | 4 | 1 | 1 | 3 | 3 | 5 | 5 | 2 |

1stpenmap 10 March, 2000

Figure 31: The full first level pentaflexagon has 5 vertices.

Flexagons constructed from regular pentagons are instructive in many respects. Unlike flexagons constructed from squares or triangles, the figure does not lie flat in the plane. Notwithstanding their nonplanarity, single cycle flexagons whose central angle sum exceeds $36o^o$ can always be run through their cycle even when they can't be laid out flat, so that the principle which bases a flexagon on a stack of polygons can always be confirmed.

Pentagon flexagons are good for observing that any number of leaves may be taken out of one pat and placed in the other. Single leaves give a figure anchored on the vertices of the pentagon, but taking two leaves gives a result which looks like tubulation, since it is anchored on the prolongations of edges surrounding the one which was skipped. With squares, that makes an exact tubulation.

Skipping three leaves forward looks like skipping two backward which is akin to folding the flexagon backwards. However, for polygons with a large number of sides, small numbers of leaves may be grasped simultaneously to advance rapidly around the cycle of faces.

The inductive part of the construction, which allows the substitution of a single polygon for an inverted stack spanning the same angle is most readily confirmed with the binary flexagon, wherein one single polygon has undergone this replacement. A stack of four pentagons is sufficiently thick as to be noticeable as an entity, and requires sufficient exertion to run through either one of the two cycles that it probably illustrates the principles of flexagons better than some other choice. A stack of two triangles is rather inconspicuous, while three squares have too much in common with coordinate axes and smooth folding to be entirely convincing.

Pentagons do nicely, avoiding the ever thicker stack which results when the number of sides of the polygon is increased. Still, binary flesagons of all orders illuminate the principles of flexagon construction.

Much of the excitement of exploring flexagons resulted from the multitude of was in which triangle flexagons as well as square flexagons could be compounded by adding new cycles to the Tuckerman traverse. Of course, all the other polygons offer the same possibilities, each time with a greatly increased scope of alternatives. A more systematic approach would be to replace *all* the polygons in the basic cycle with inverted stacks, arriving at what one might call the second level of flexagon. Working along similar lines would lead to third level flexagns, fourth level, and so on. All can be prepared from winding up previously prepared polygon strips, but the thickness of a paper implementation rapidly makes physical realization difficult and then impossible.

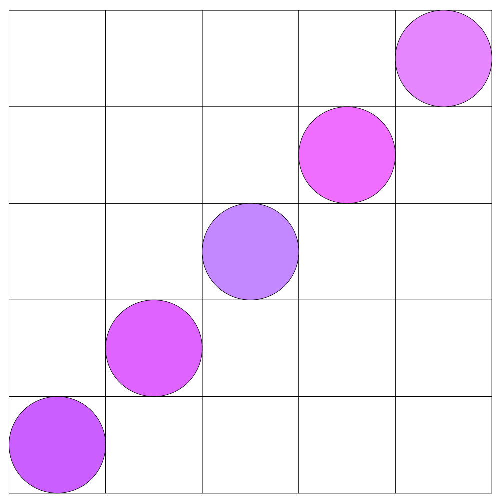


Figure 32: Permutation of the pentagons along the strip for a first level flentagon.

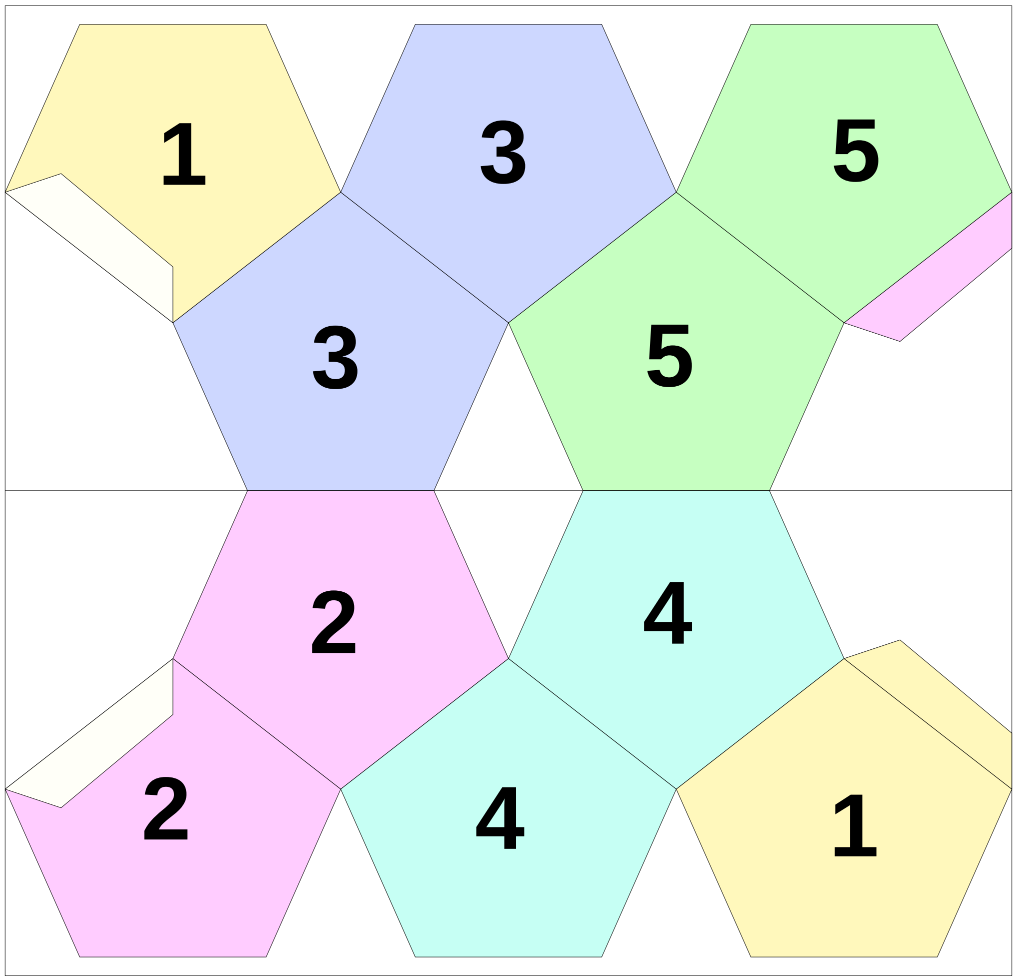



Figure 33: Top side of a flentagon consisting of a single cycle. The figure should be cut horizontally through the middle, and one tab pasted to match colors. Once the figure has been folded up, the other tab should be pasted where space for it has been provided. The flexagon will not lie flat, but the two sectors will divide easily into twp pats each, within which leaves can be separated and moved from one pat to the other by flexing.

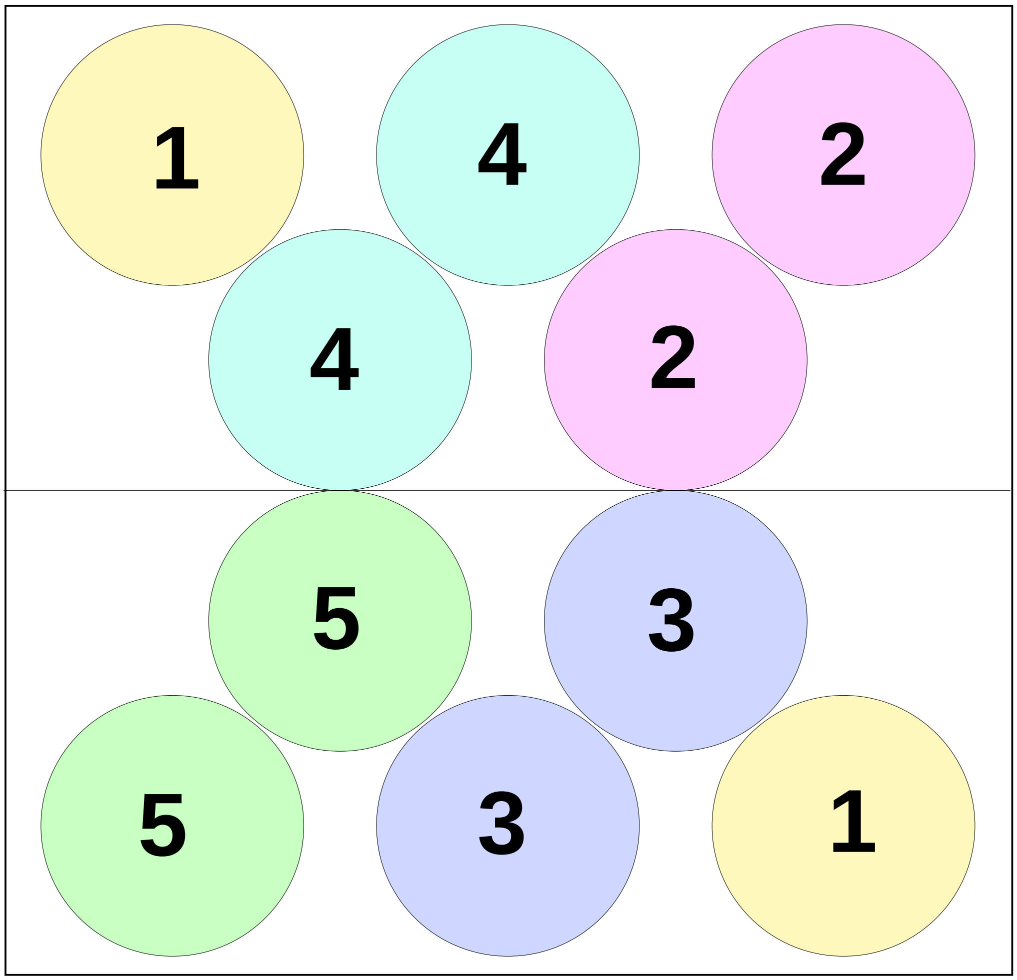



Figure 34: Bottom side of a flentagon consisting of a single cycle.

# 8 Binary Flentagon

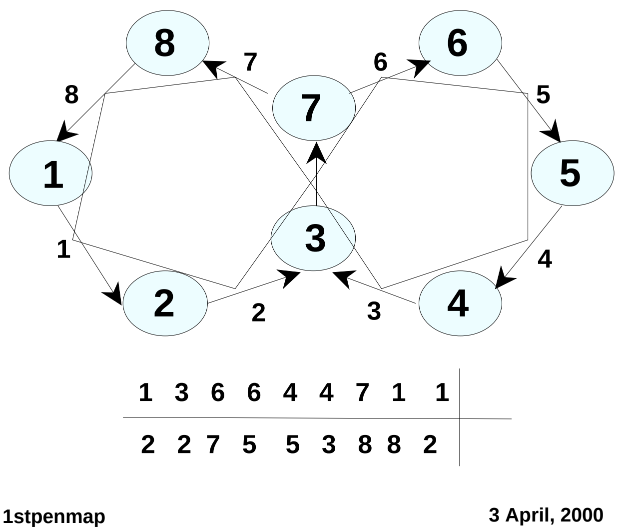



Figure 35: The binary pentaflexagon has two cycles, each of which has two vertices in common with the other one, for a total of eight.

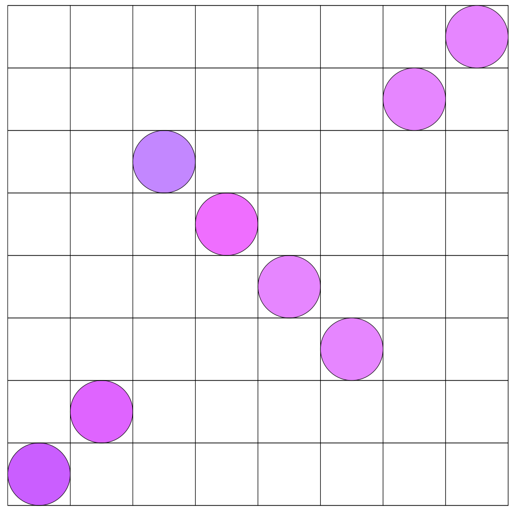


Figure 36: Permutation of the pentagons along the strip for a binary pentaflexagon.

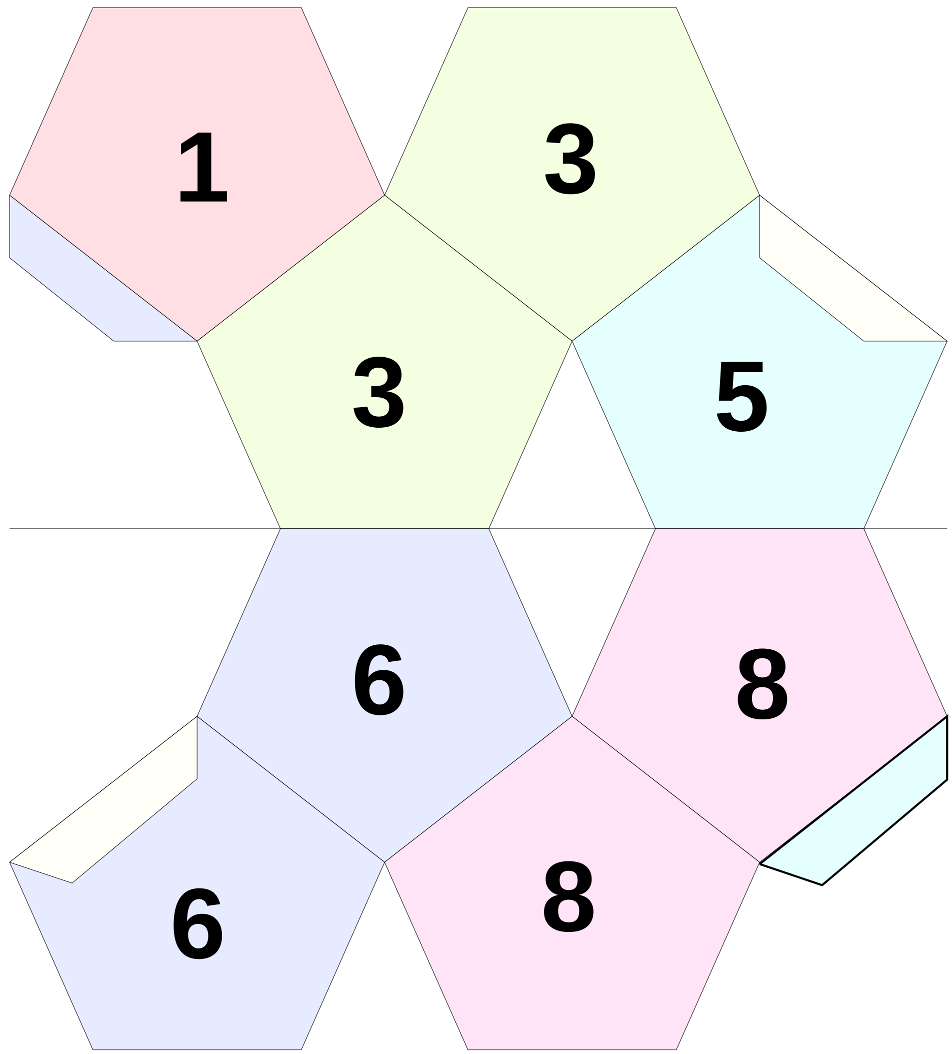



Figure 37: Top side of the first level pentaflexagon cutout. The flexagon has ten faces, so this cutout provides material for just one of the sectors needed for the flexagon.

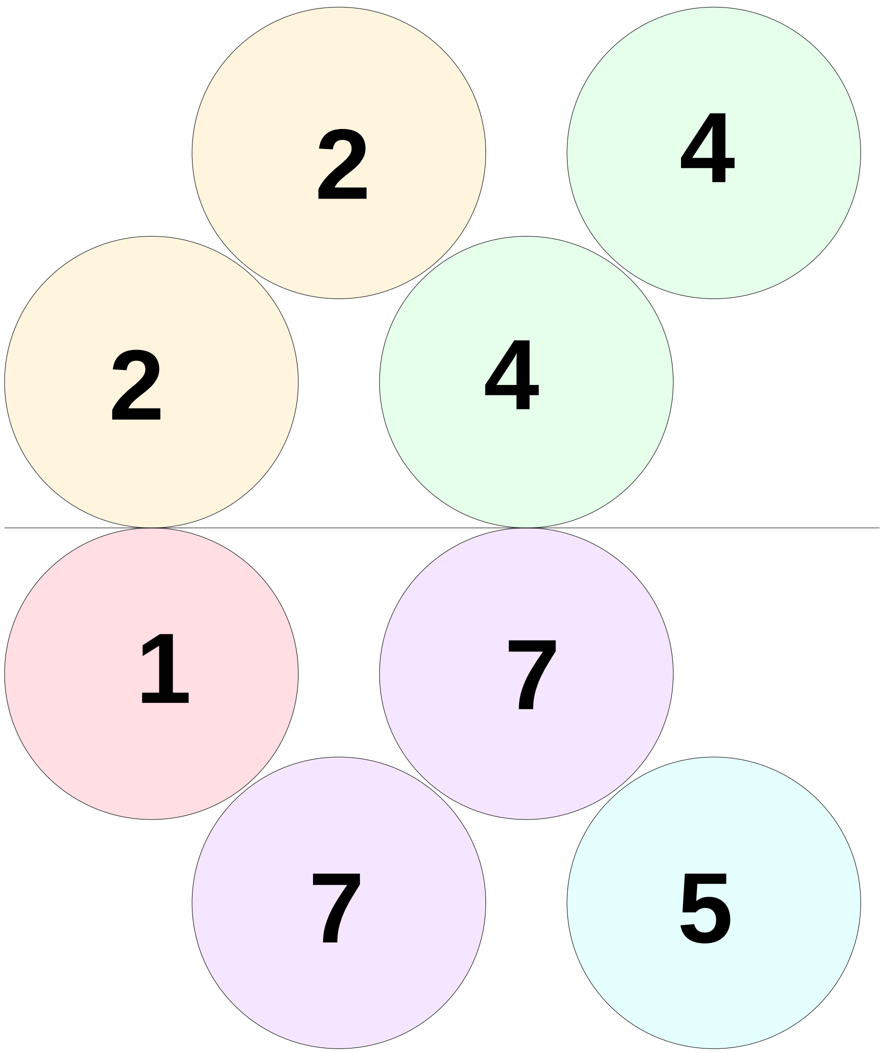



Figure 38: Bottom side of the first level binary pentaflexagon cutout.

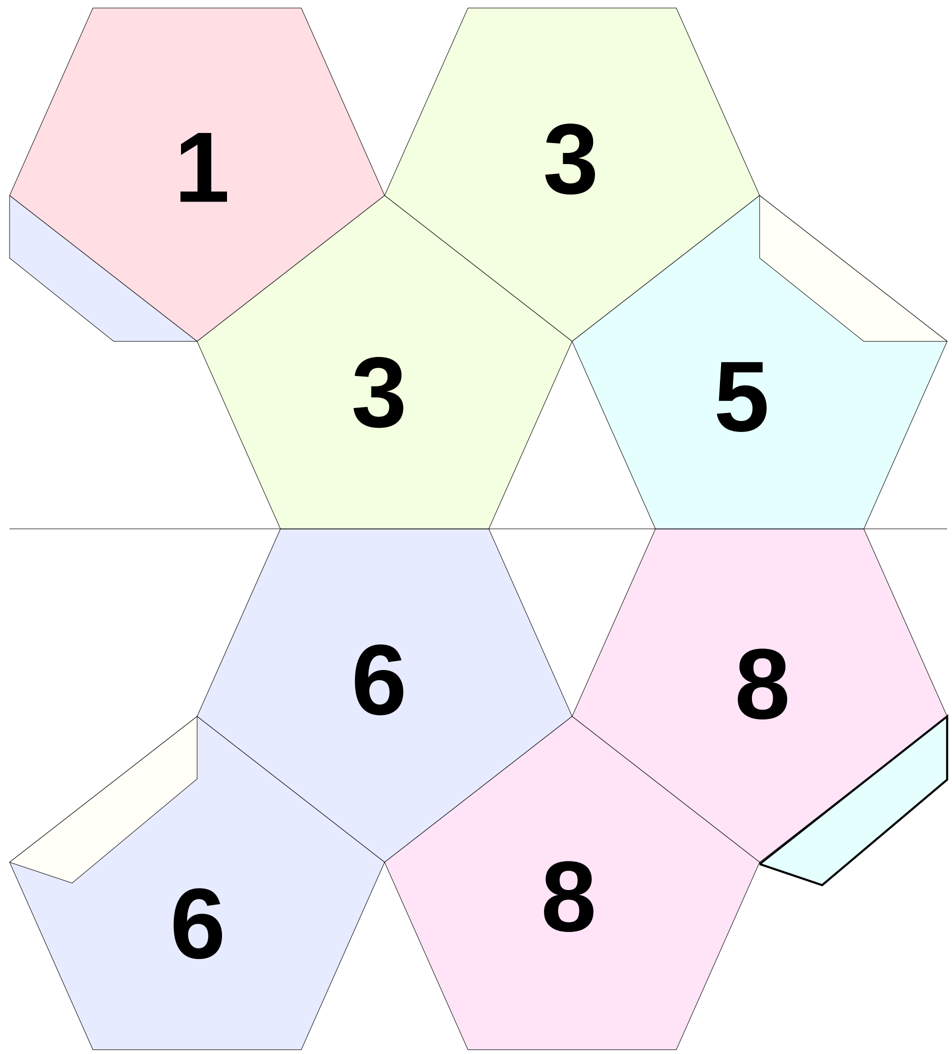



Figure 39: Copy of the top side of the first sector of the binary flentagon, for making the second sector.

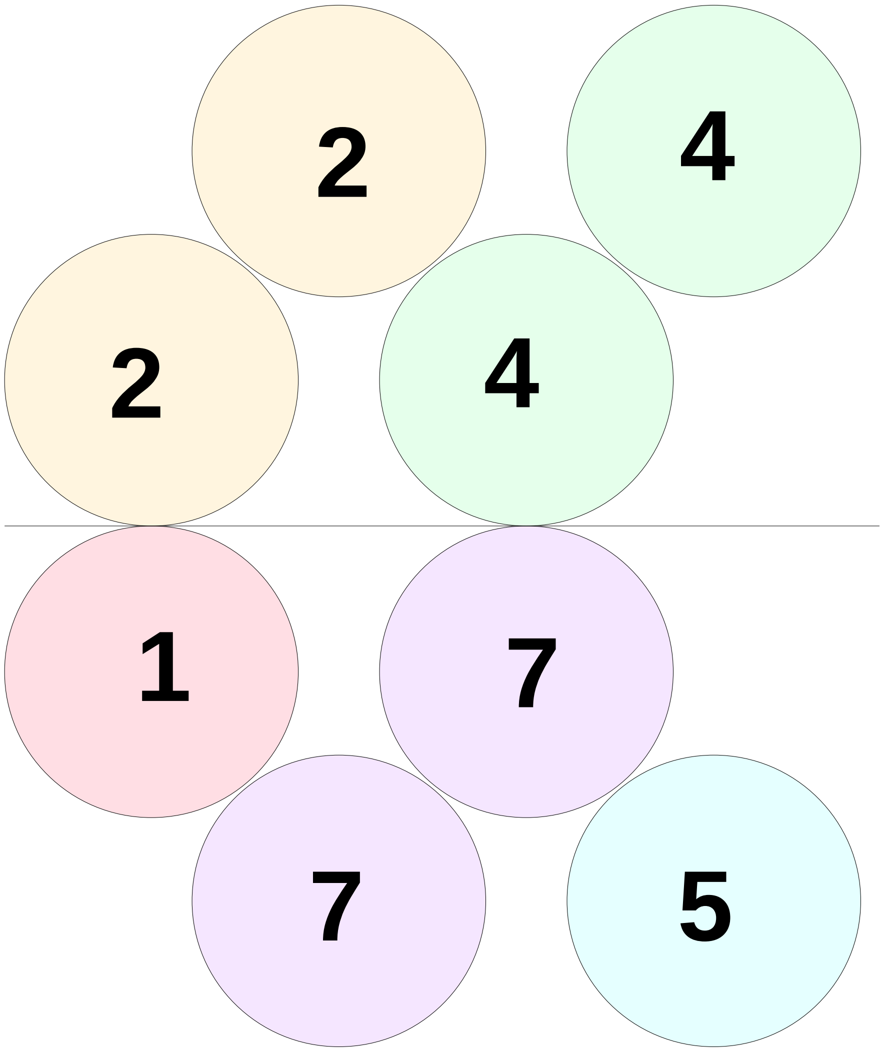



Figure 40: Copy of the bottom side of the first sector of the binary flentagon, for making the second sector.

# 9 Second Level Flentagon

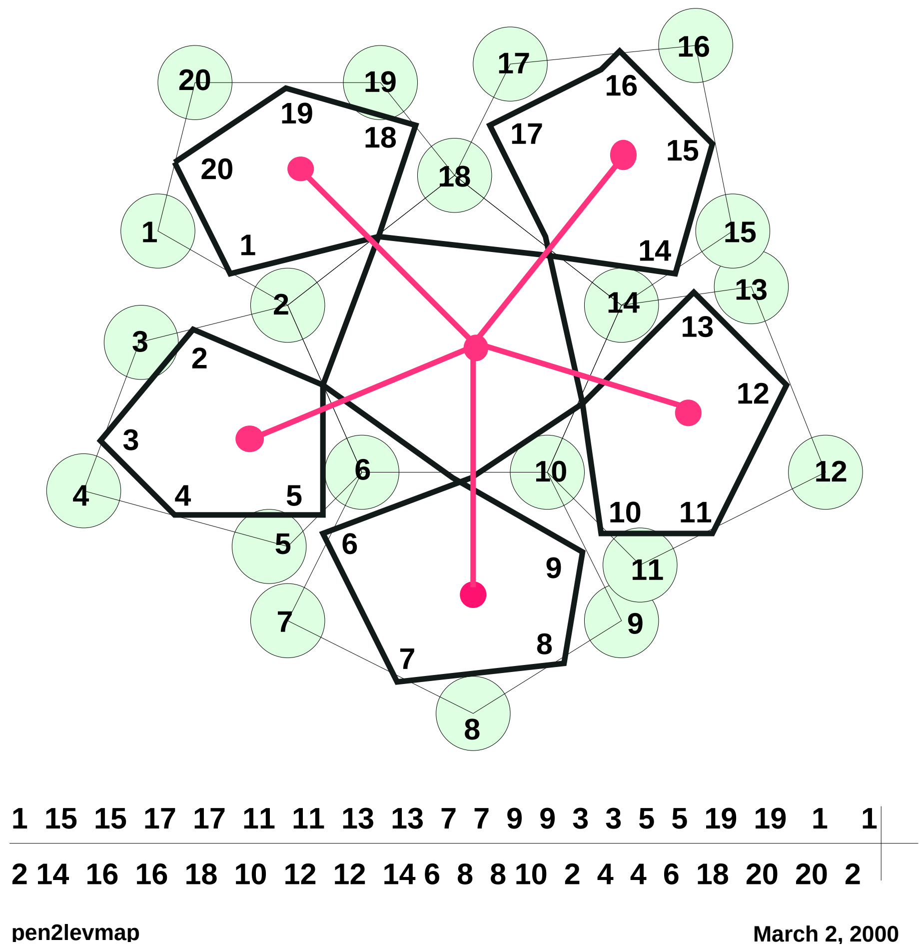



Figure 41: Since each edge of the first level flentagon spawns three new vertices, the full second level flentagon has 20 vertices.

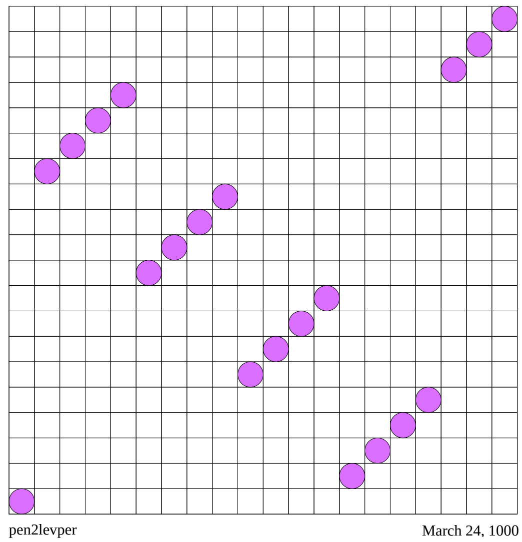



Figure 42: Permutation of the pentagons along the strip for a second level flentagon.

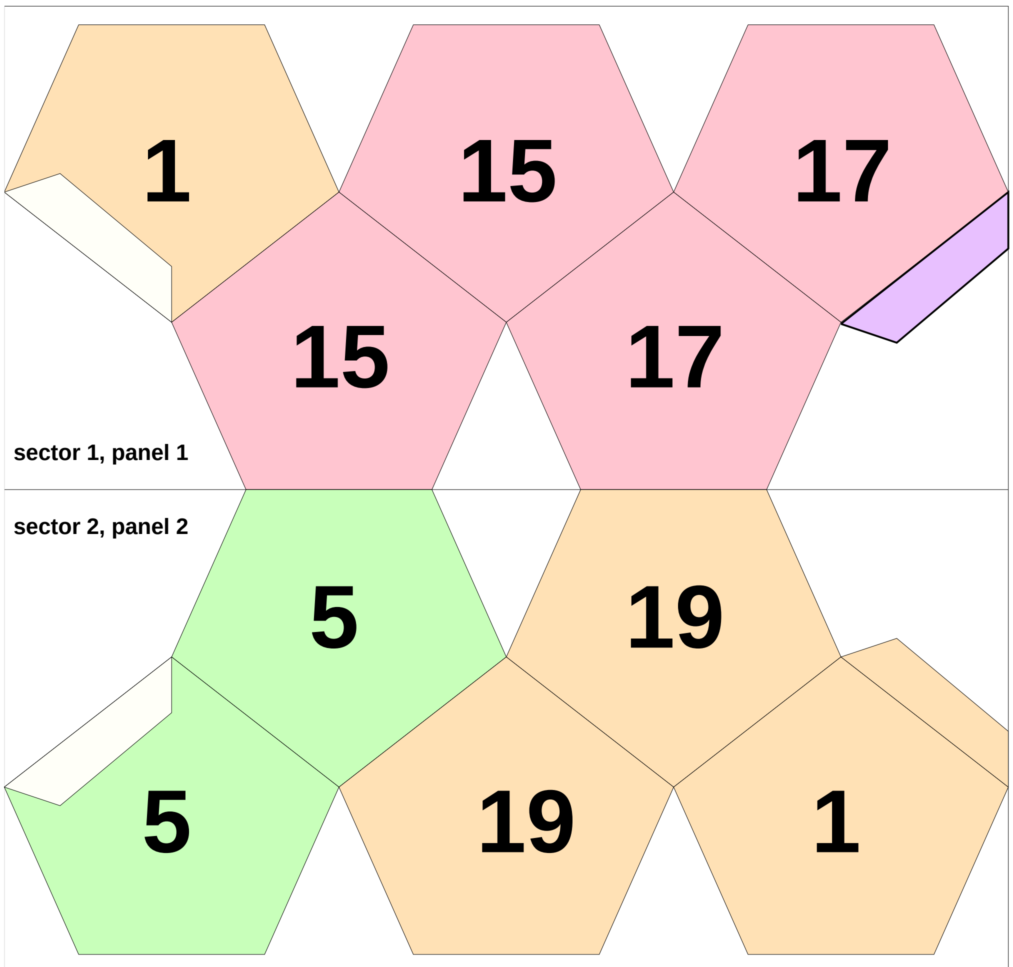



Figure 43: Top side of a segment which can be cut out, and by using four copies altogether, the strip for a second level flentagon can be constructed. This is the first copy.

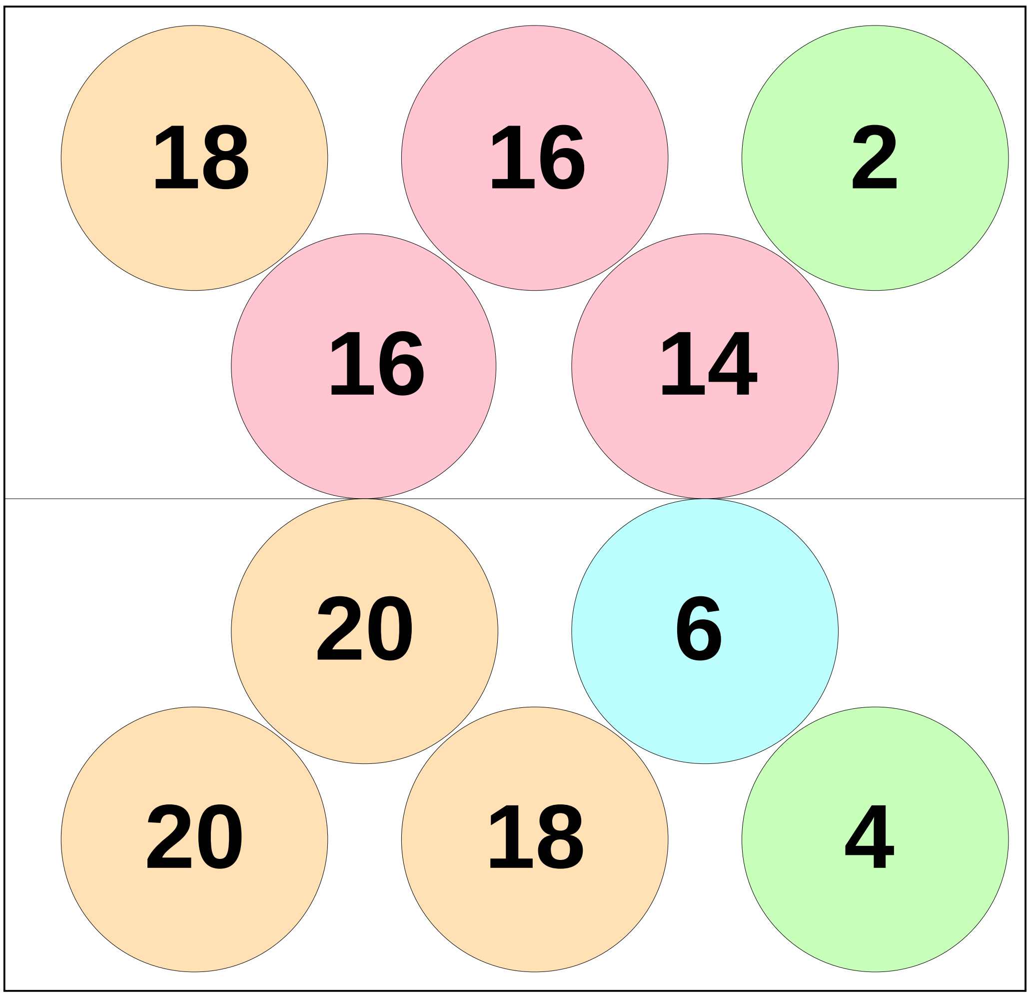



Figure 44: Bottom side of a second order flentagon, first copy of four.

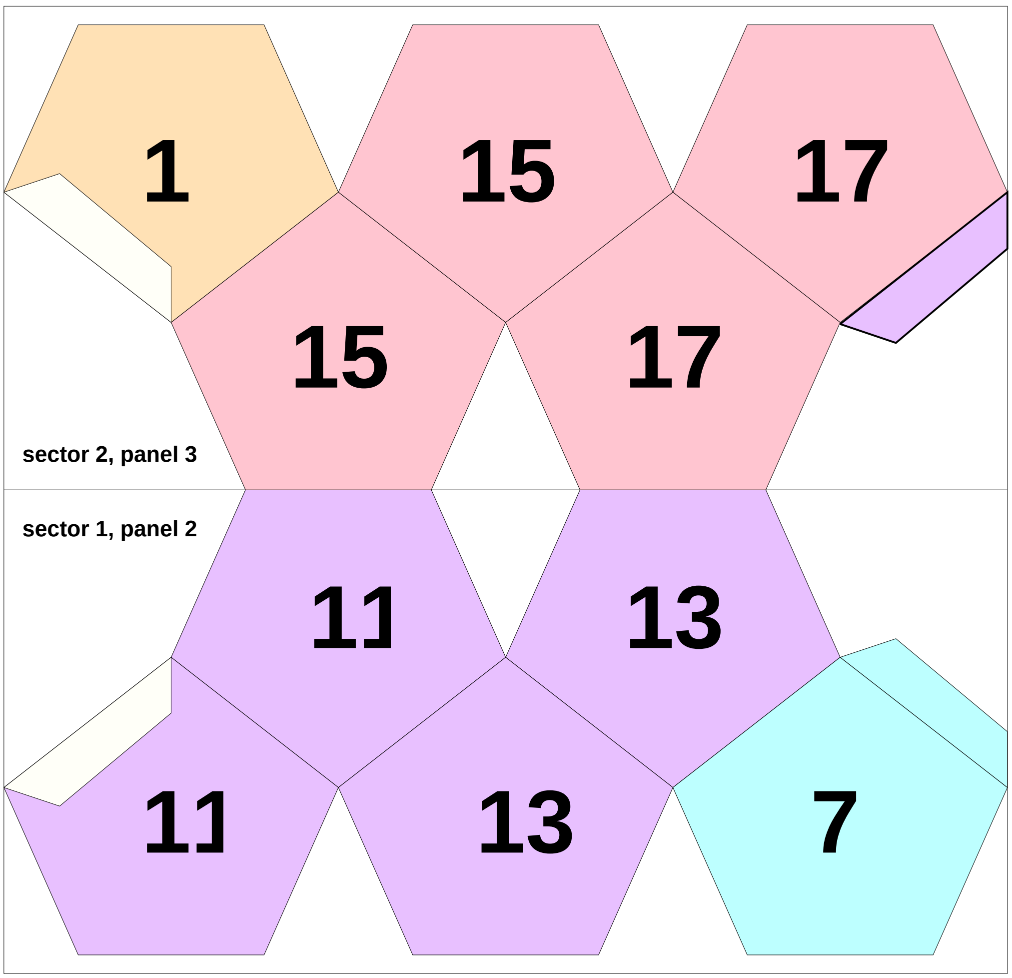



Figure 45: Top side of a segment which can be cut out, and by using four copies altogether, the strip for a second level flentagon can be constructed. This is the second copy.

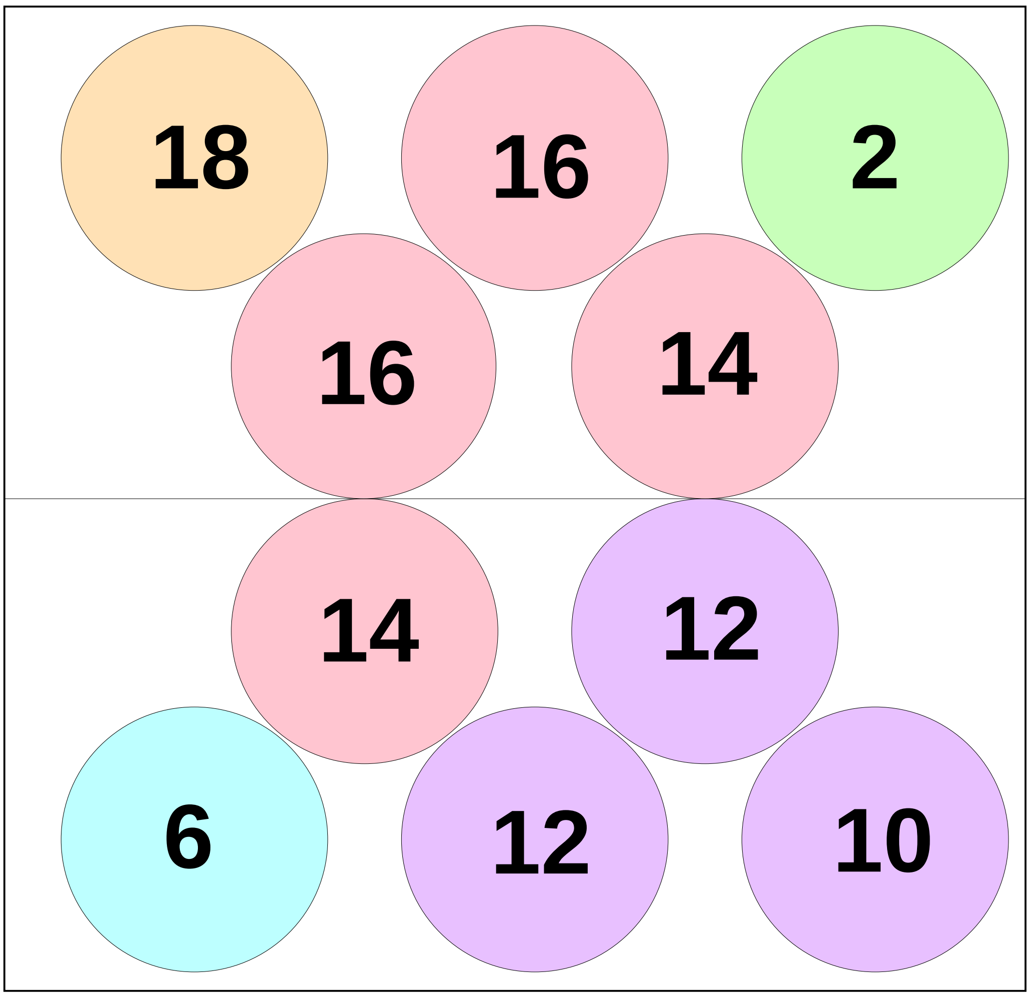



Figure 46: Bottom side of a second order flentagon, second copy of four.

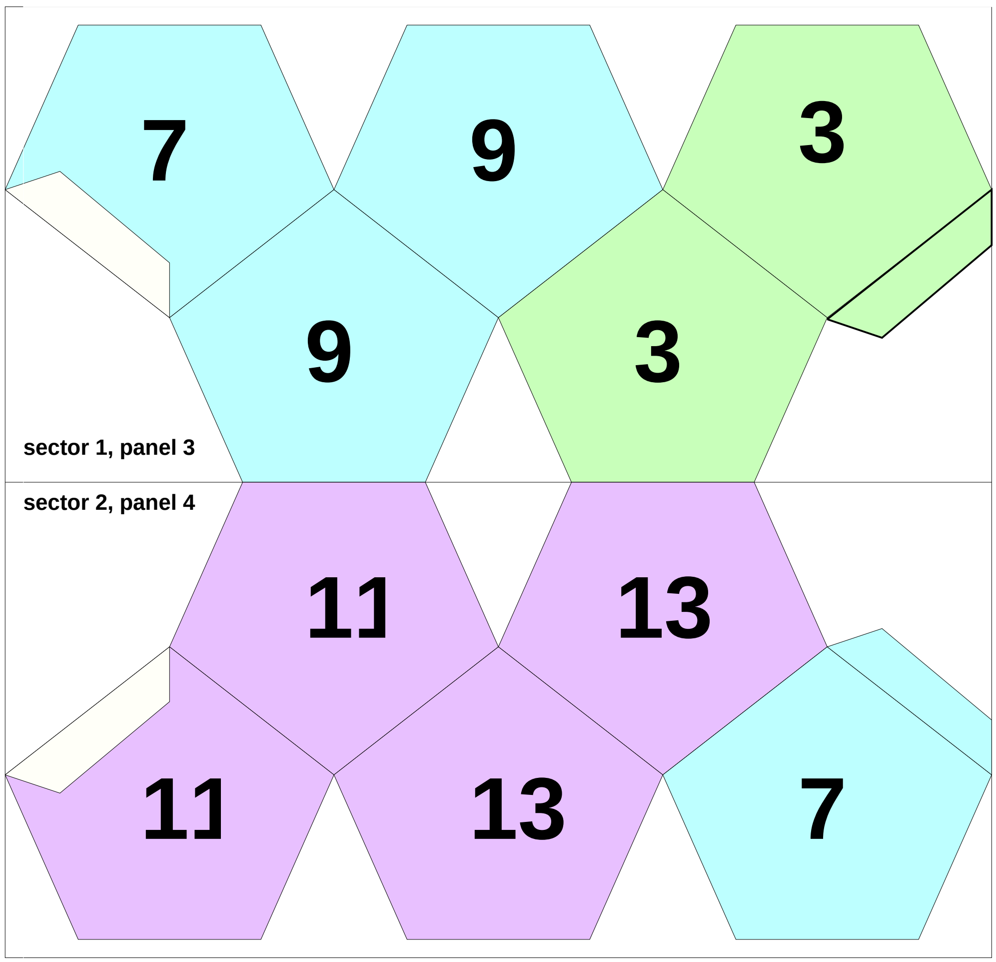



Figure 47: Top side of a segment which can be cut out, and by using four copies altogether, the strip for a second level flentagon can be constructed. This is the third copy.

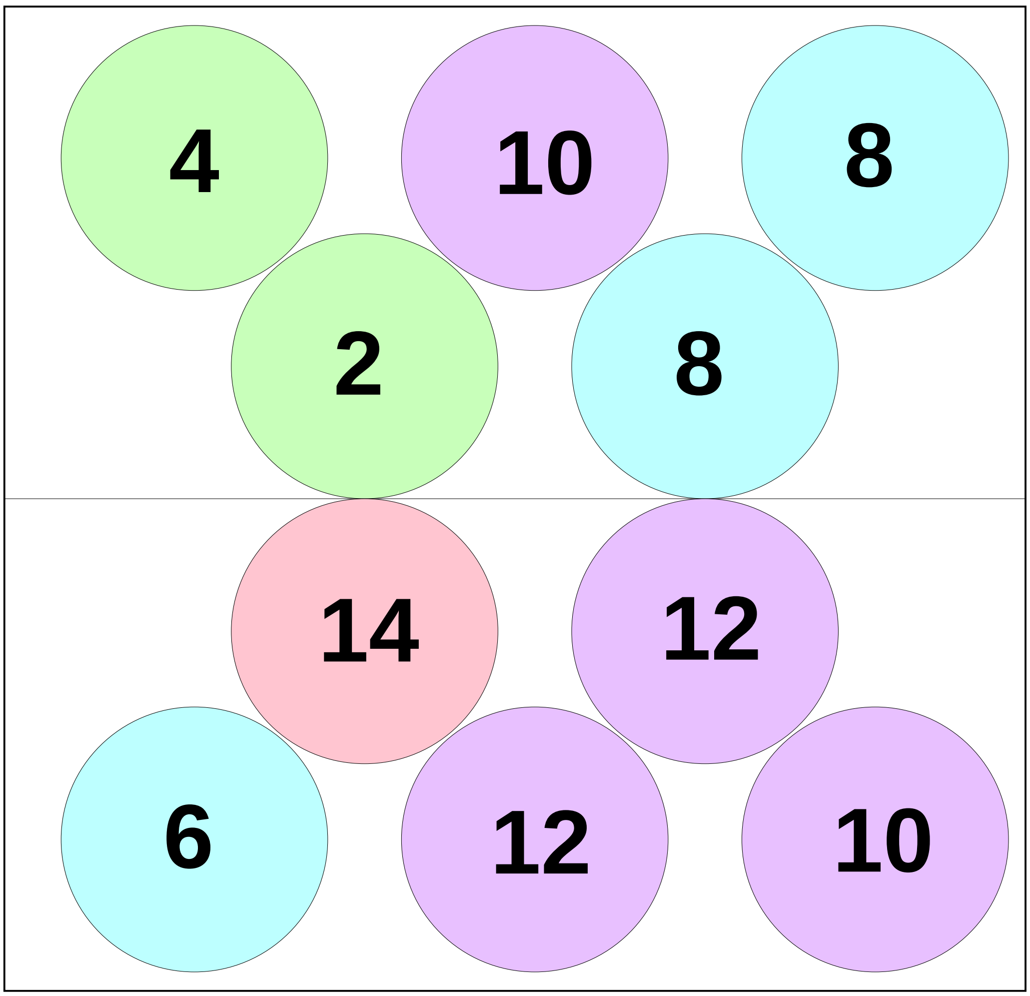



Figure 48: Bottom side of a second order flentagon, third copy of four.

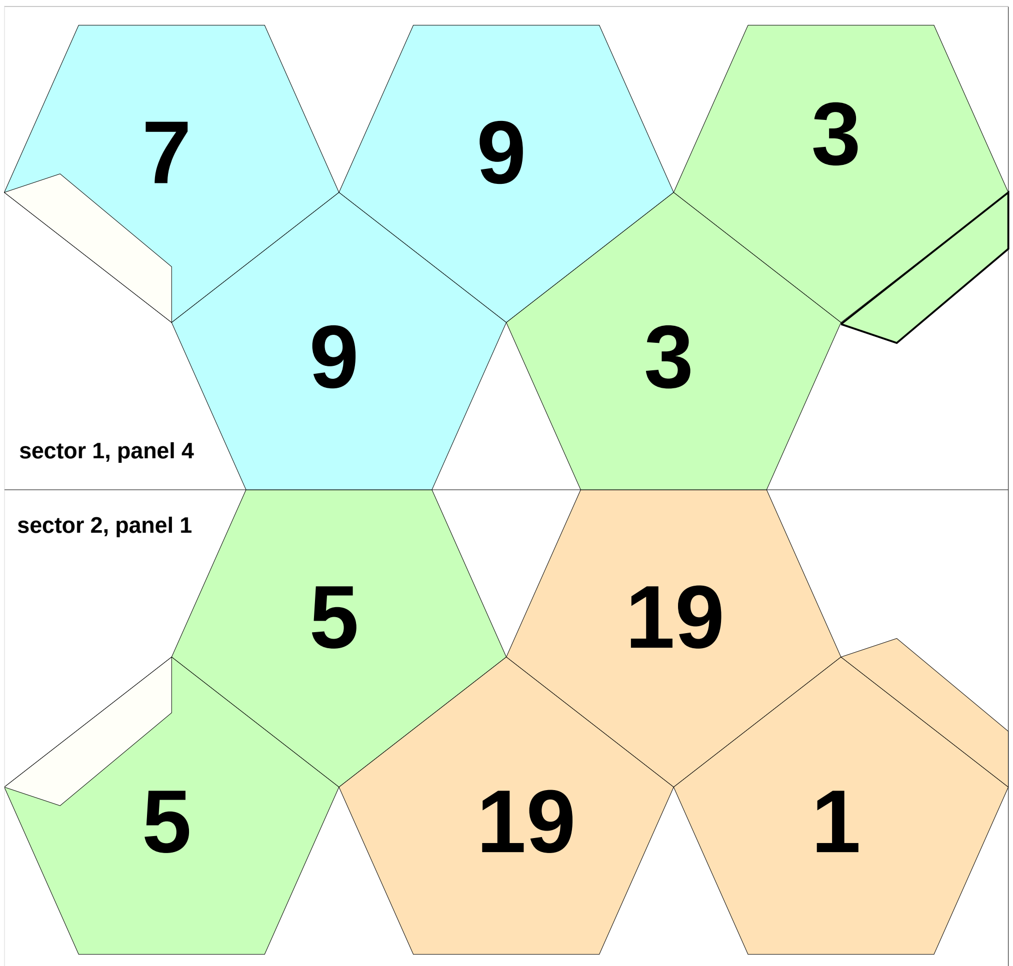



Figure 49: Top side of a segment which can be cut out, and by using four copies altogether, the strip for a second level flentagon can be constructed. This is the fourth copy.

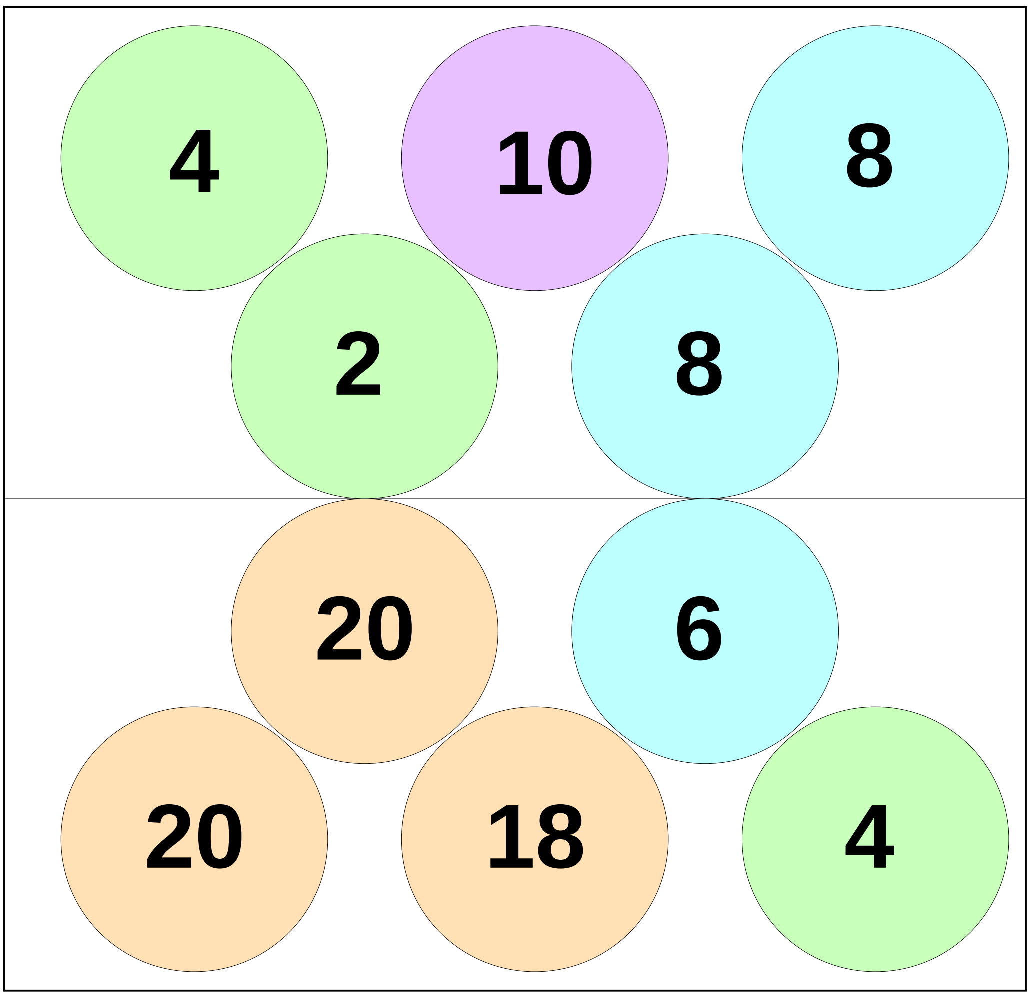



Figure 50: Bottom side of a second order flentagon, fourth copy of four.

# 10 First Level Flessagon

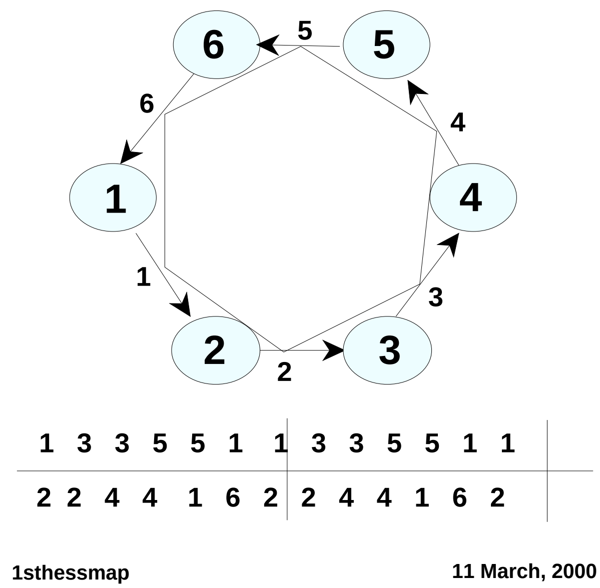



Figure 51: The first level hessaflexagon has 6 vertices.

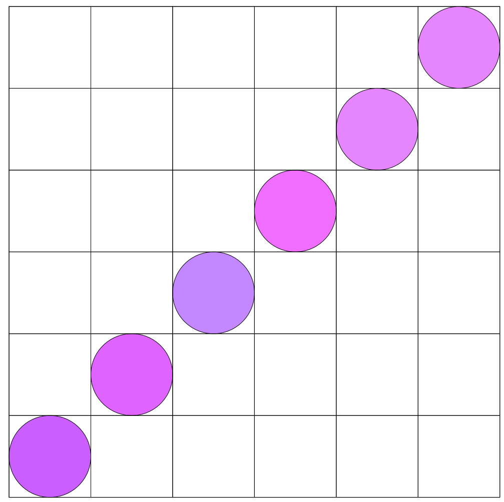


Figure 52: Permutation of the hexagons along the strip for a first level flessagon.

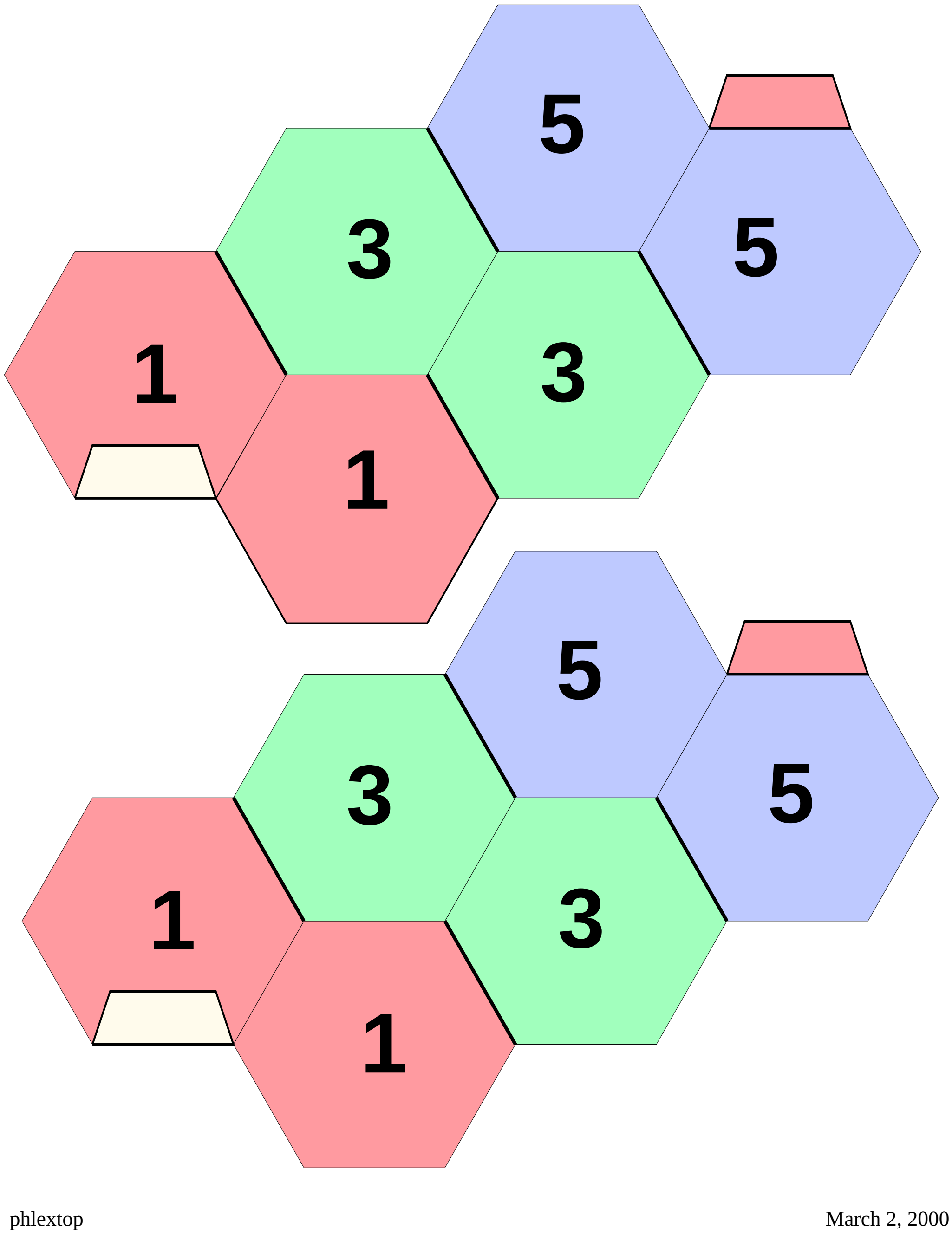



Figure 53: Top side of a flessagon consisting of a single cycle.

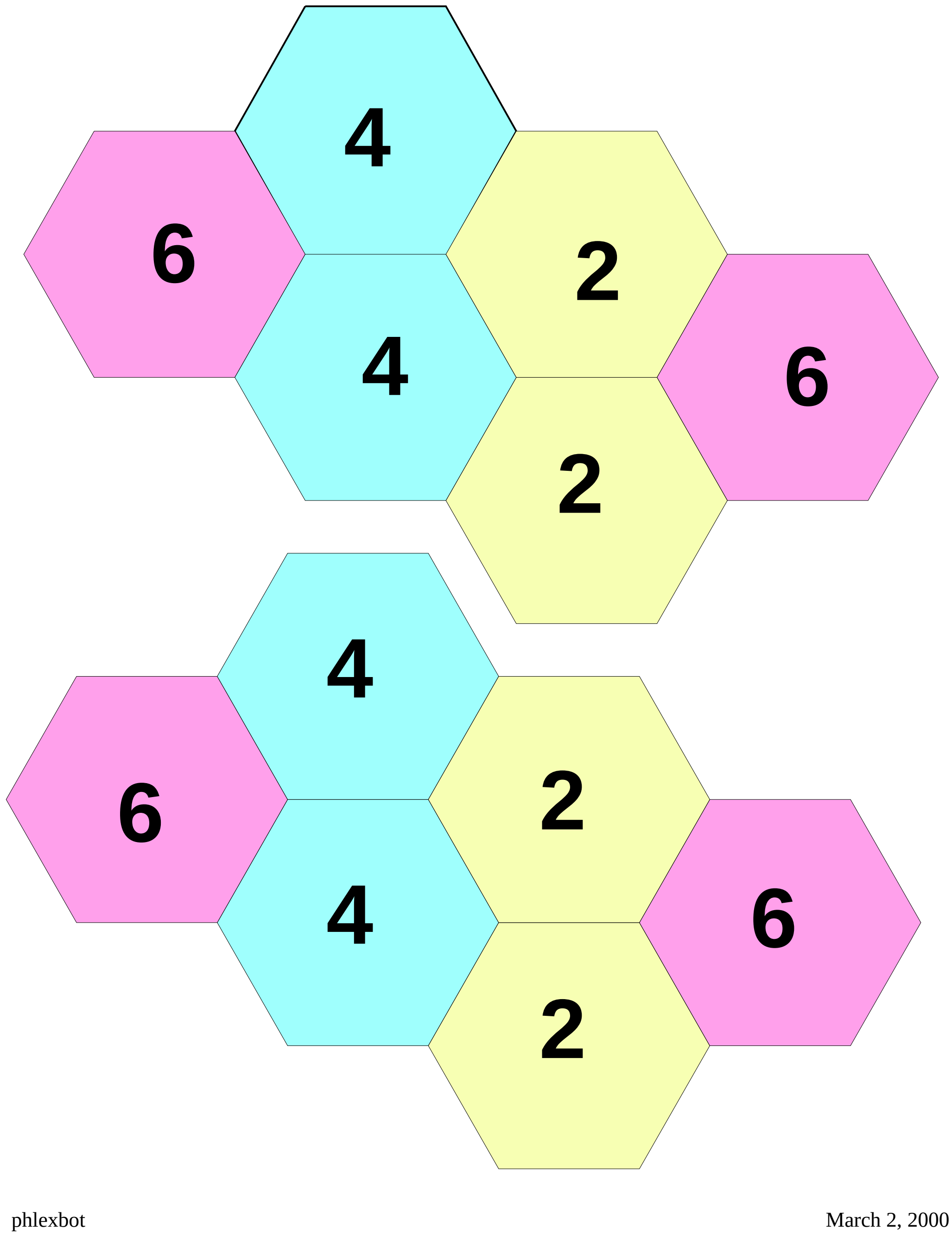



Figure 54: Bottom side of a flessagon consisting of a single cycle.

# 11 Binary Flessagon

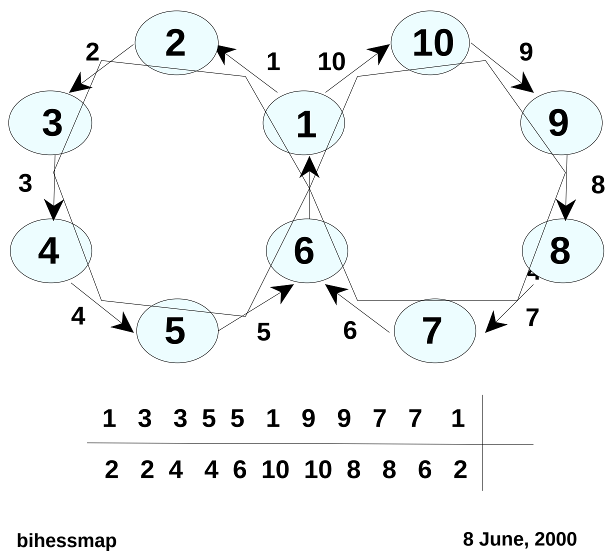



Figure 55: The binary hessaflexagon has two cycles, each of which has two vertices in common with the other one, for a total of ten.

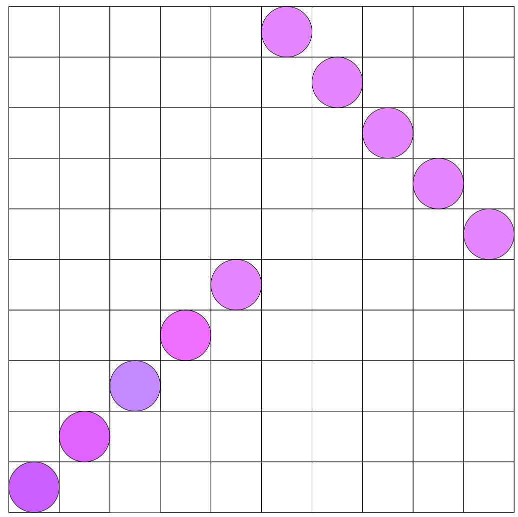


Figure 56: Permutation of the hexagons along the strip for a binary hessaflexagon.

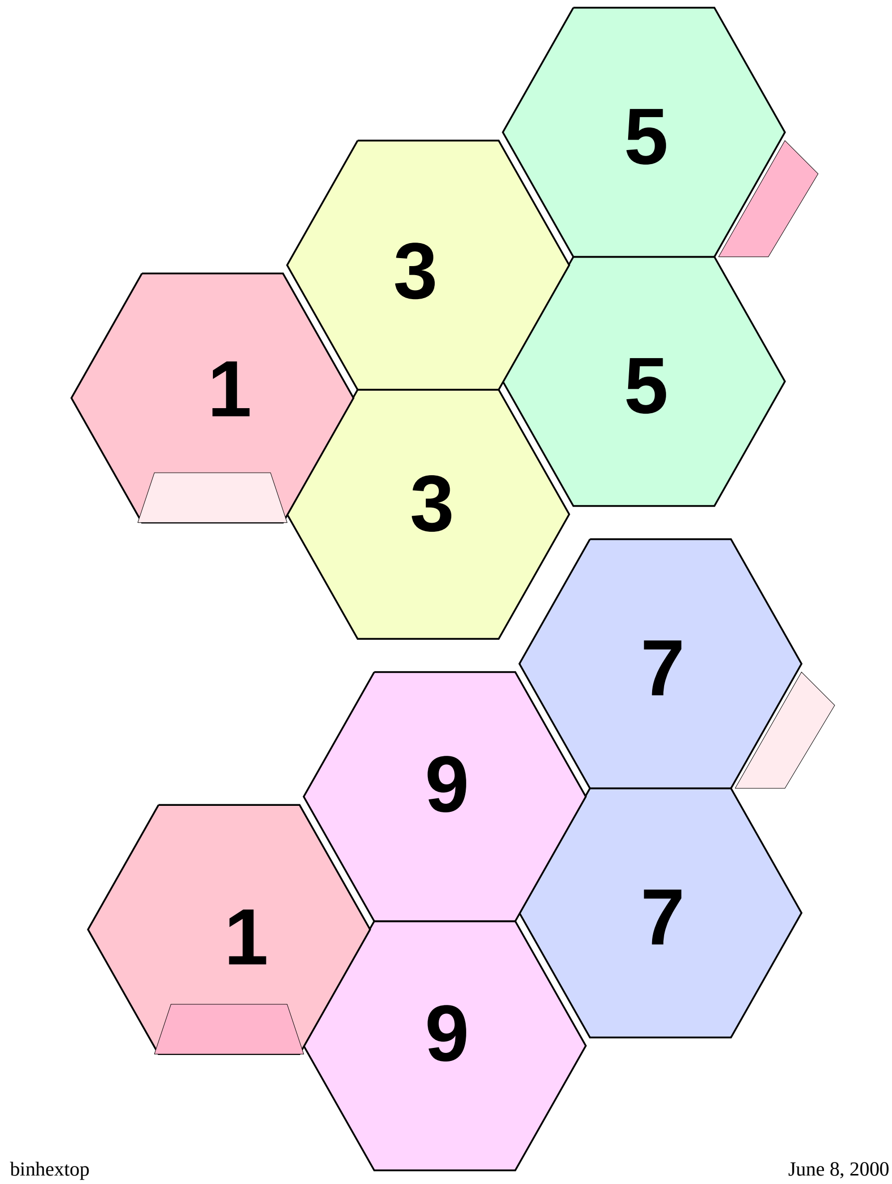



Figure 57: Top side of the first level hessaflexagon cutout. With ten faces, this cutout provides material for just the first sector (composed of two pats) of the two needed for the flexagon.

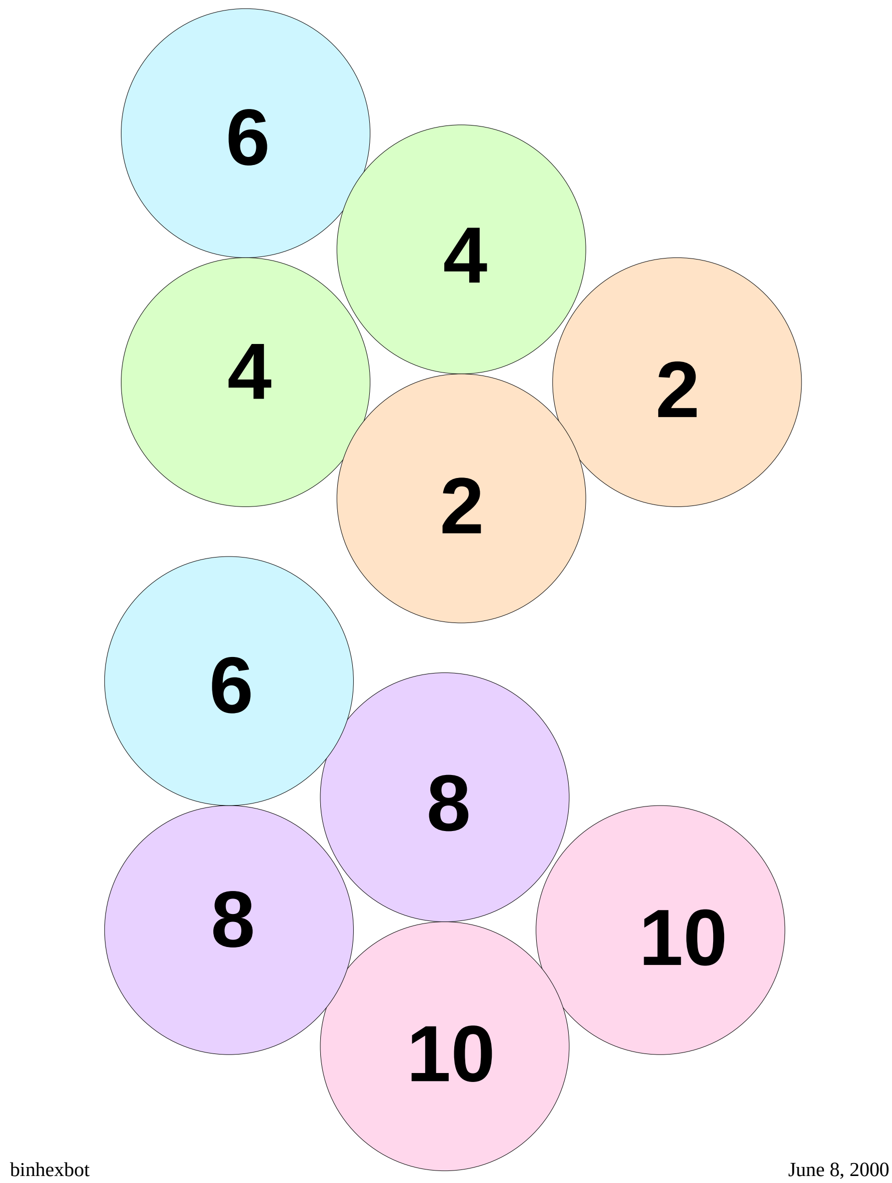



Figure 58: Bottom side of the first sector of the binary hessaflexagon cutout. Two sectors are required.

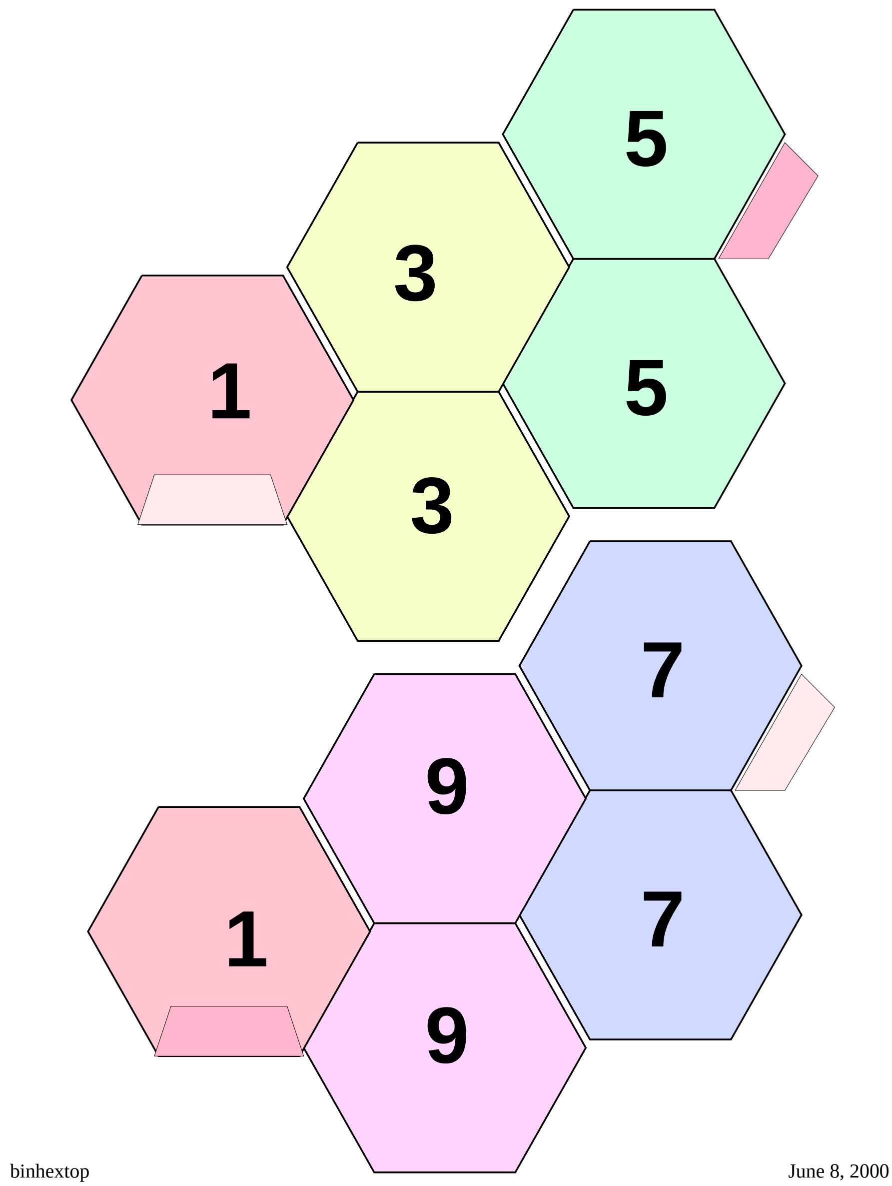



Figure 59: Top side of the first level hessaflexagon cutout. With ten faces, this cutout provides material for just the second sector (composed of two pats) of the two needed for the flexagon.

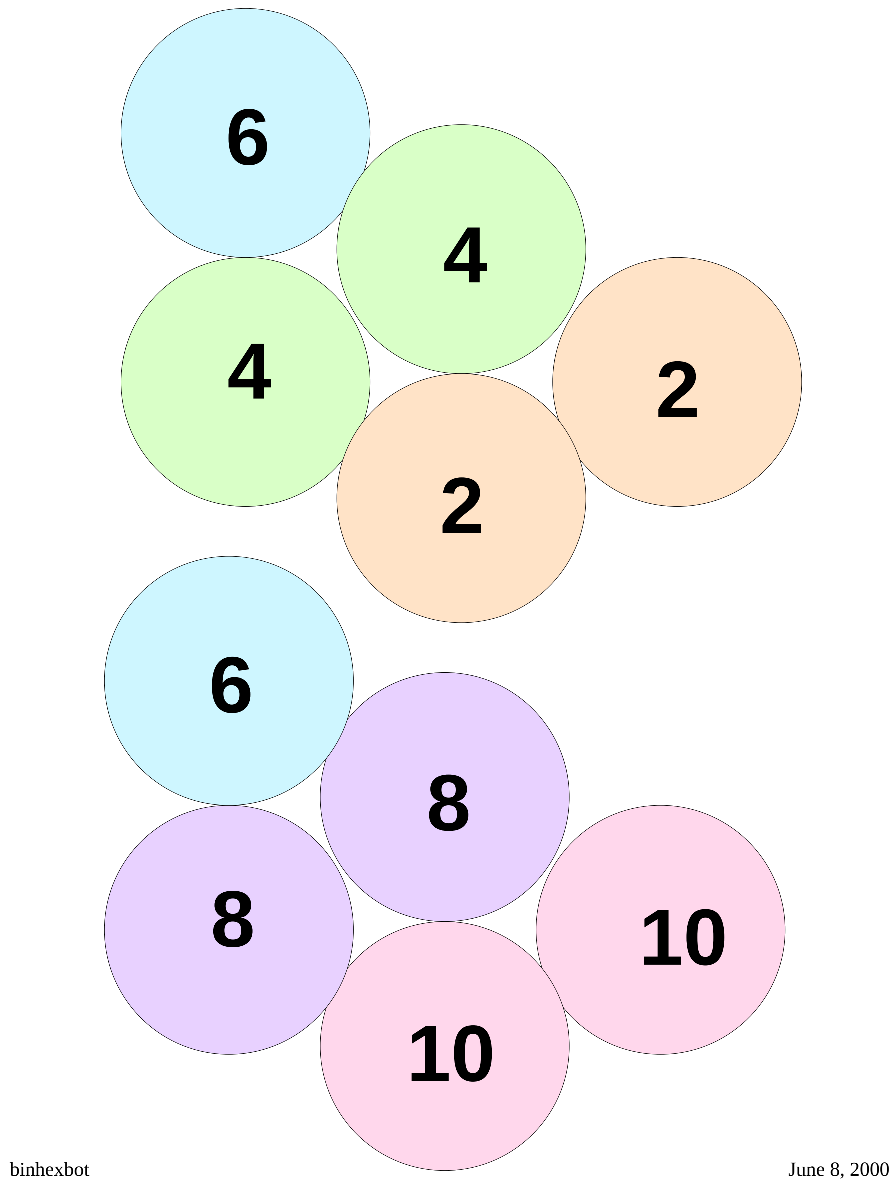



Figure 60: Bottom side of the second sector of the binary hessaflexagon cutout. Two sectors are required.

# 12 Second Level Flessagon

## 12.1 Tukey hexagons

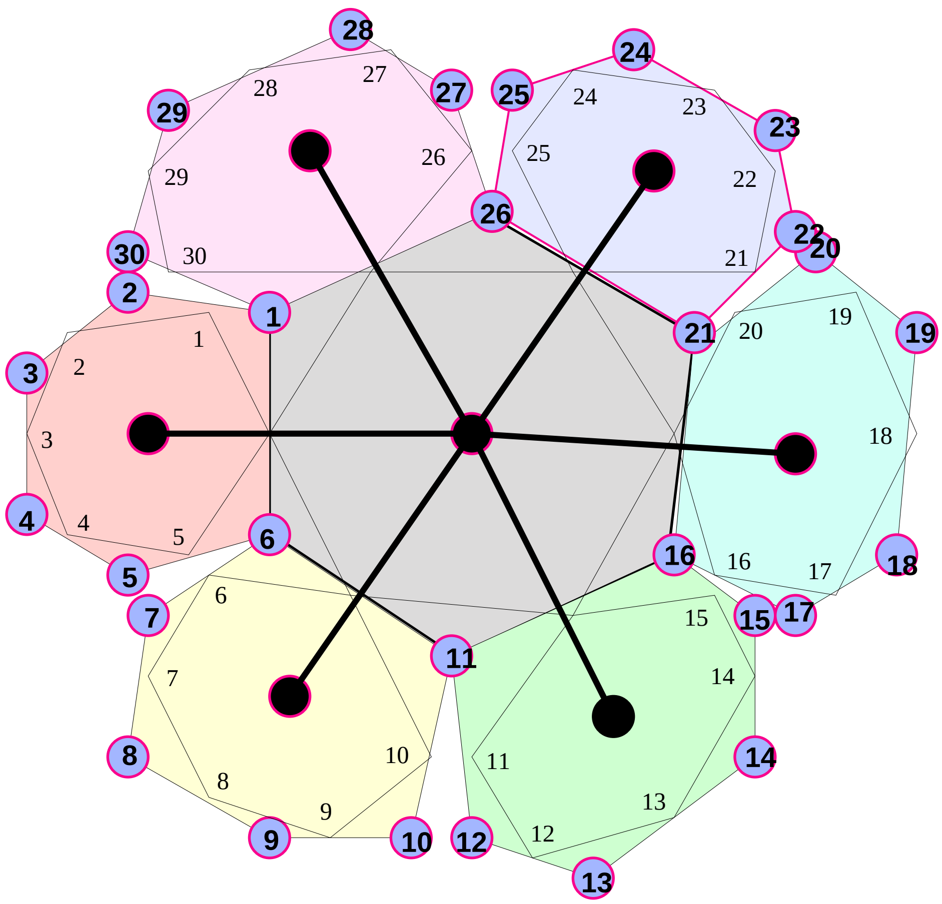


Figure 61: Since each of the six edges of the first level flessagon spawns four new vertices, the full second level flessagon has 30 vertices since $(4 * 6 + 6 = 30)$.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| 1 | 3 | 3 | 5 | 5 | 27 | 27 | 29 | 29 | 1 | 21 | 23 | 23 | 25 | 25 | 17 | 17 | 19 | 19 | 21 | 11 | 13 | 13 | 15 | 15 | 7 | 7 | 9 | 9 | 11 | 1 |
| 2 | 2 | 4 | 4 | 6 | 26 | 28 | 28 | 30 | 30 | 22 | 22 | 24 | 24 | 26 | 16 | 18 | 18 | 20 | 20 | 12 | 12 | 14 | 14 | 16 | 6 | 8 | 8 | 10 | 10 | 2 |

If the strips in Figures 63 through 71 had been displayed as six pages with strings of five hexagons each, there would have been an artistic parity problem, but the structure of the second order flexagon would have been that much clearer. Anyway, once the strips have been cut out, pasted, and made ready for folding, the second order periodicity is evident enough.

The reason for this is that any $n$-gon in a regular flexagon can be replaced by a fanfolded strip of $n-1$ $n$-gons turned upside down while still making the same connections as before. When all the original $n$-gons have been so replaced, the next higher order of flexagon results. Treating these subpats as units, everything remains as before; but each of them can be opened up via the mountain-valley transition (pinching), to get cycles based just on the subpat.

## 12.2 Second level permutations

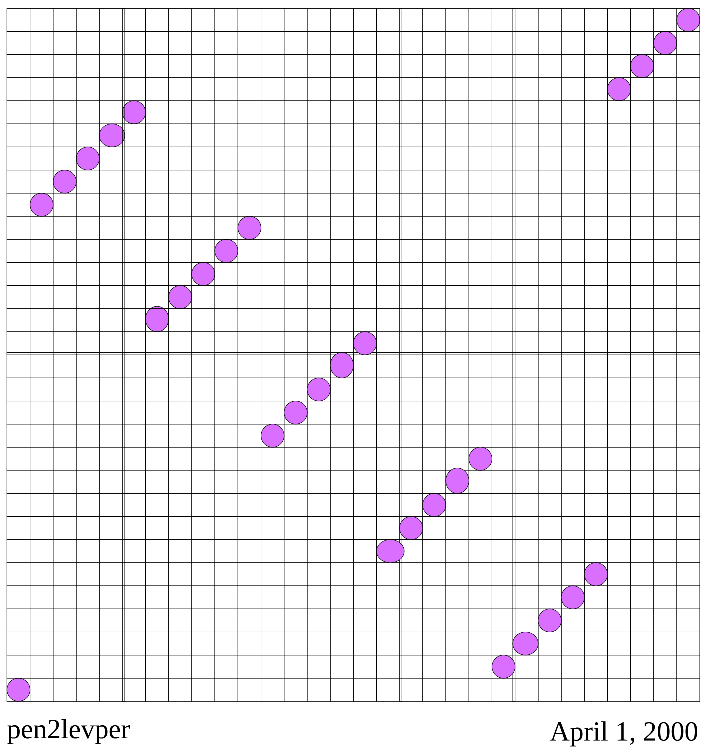



Figure 62: Permutation of the hexagons along the strip for a second level flessagon.

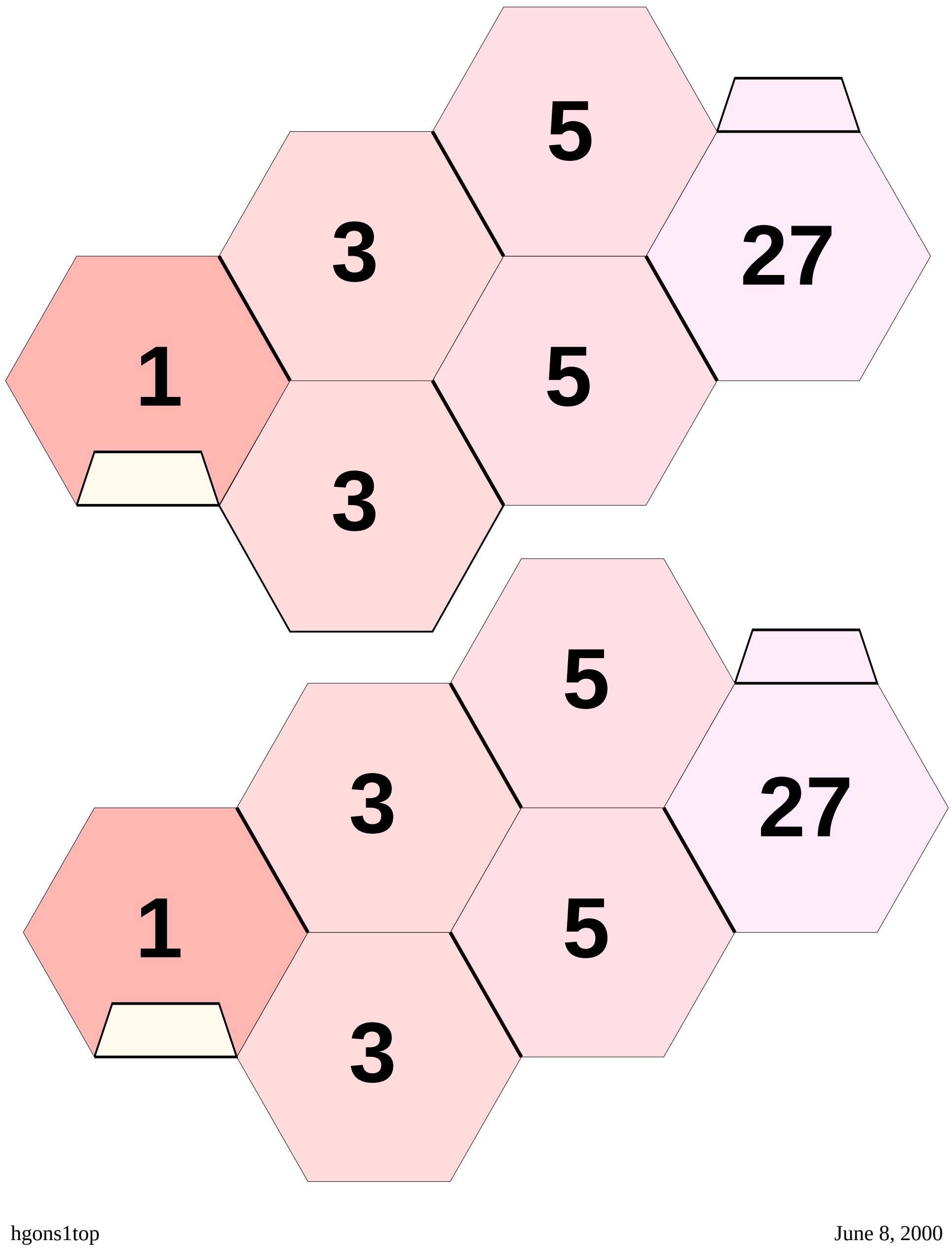



Figure 63: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level flessagon can be constructed. This is the first copy.

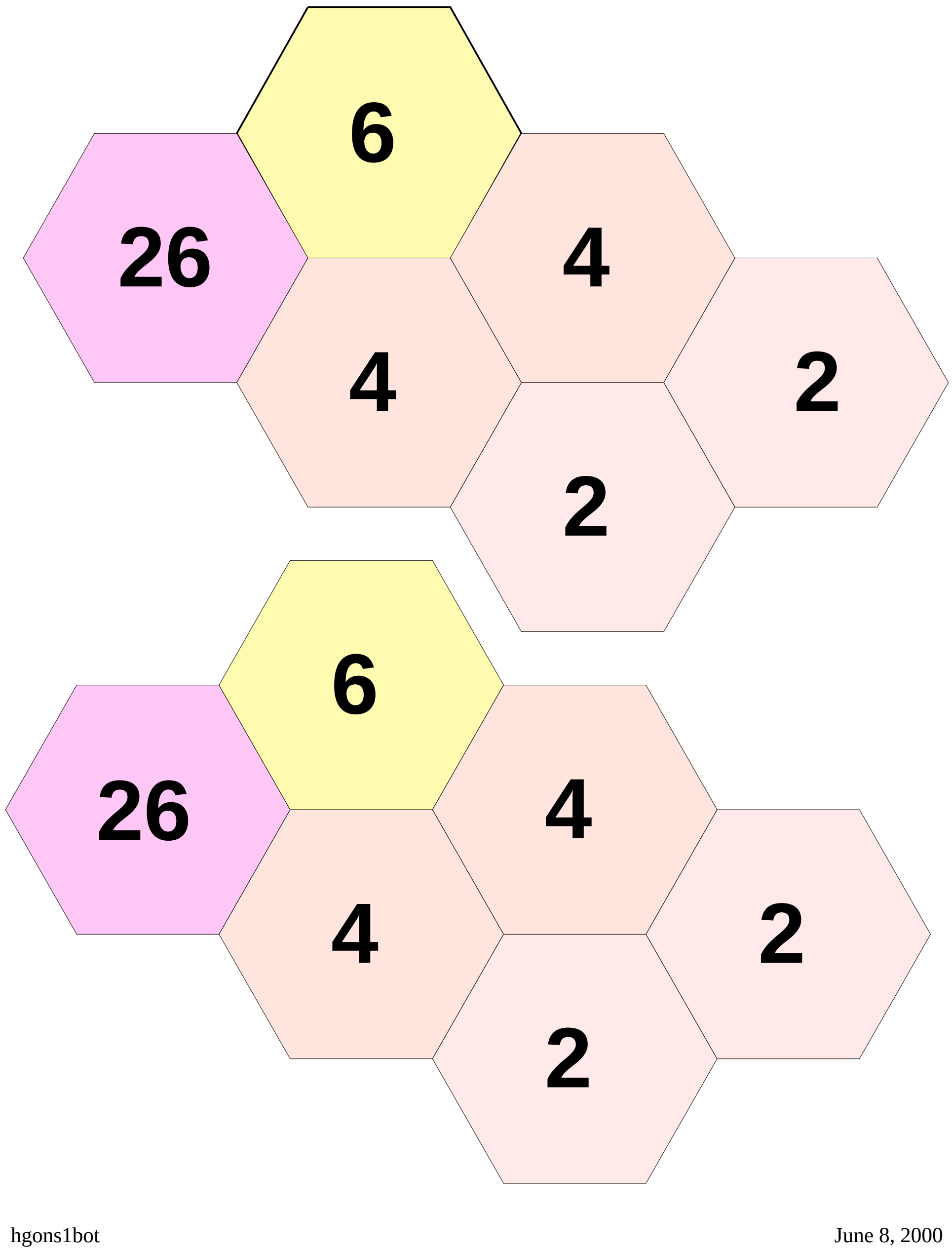



Figure 64: Bottom side of a second order flessagon.

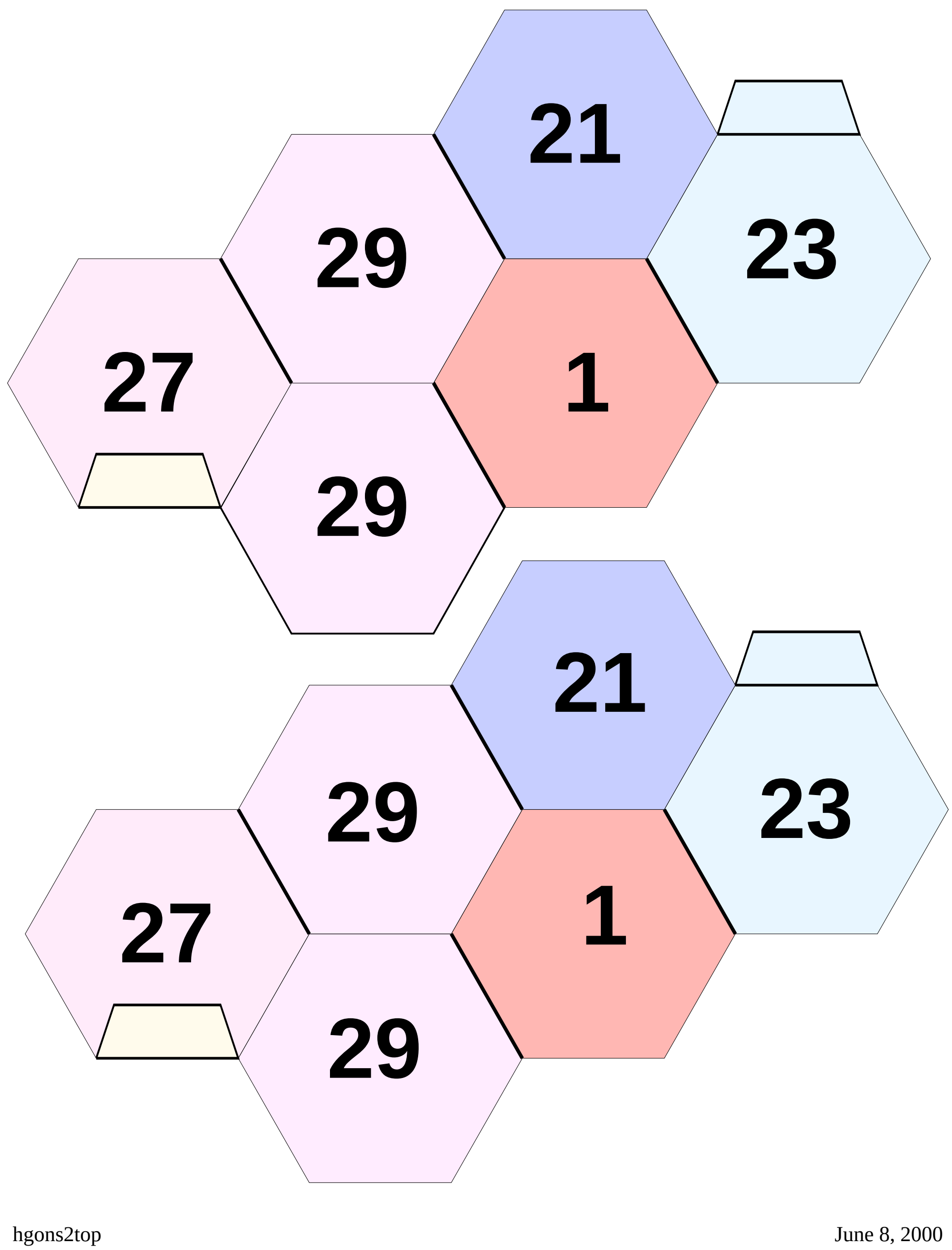
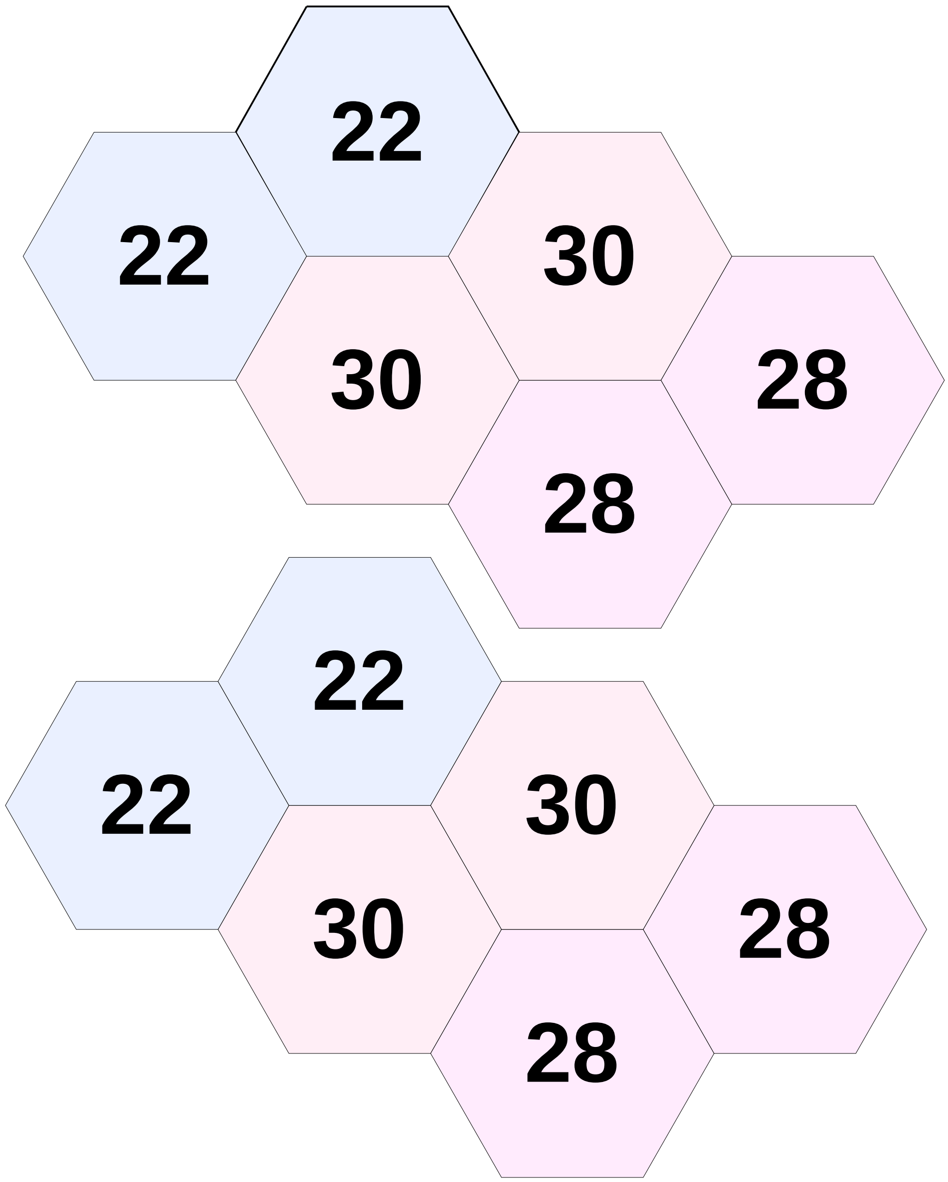



Figure 65: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level flessagon can be constructed. This is the second copy.




hgons2bot June 8, 2000

Figure 66: Bottom side of a second order flessagon.

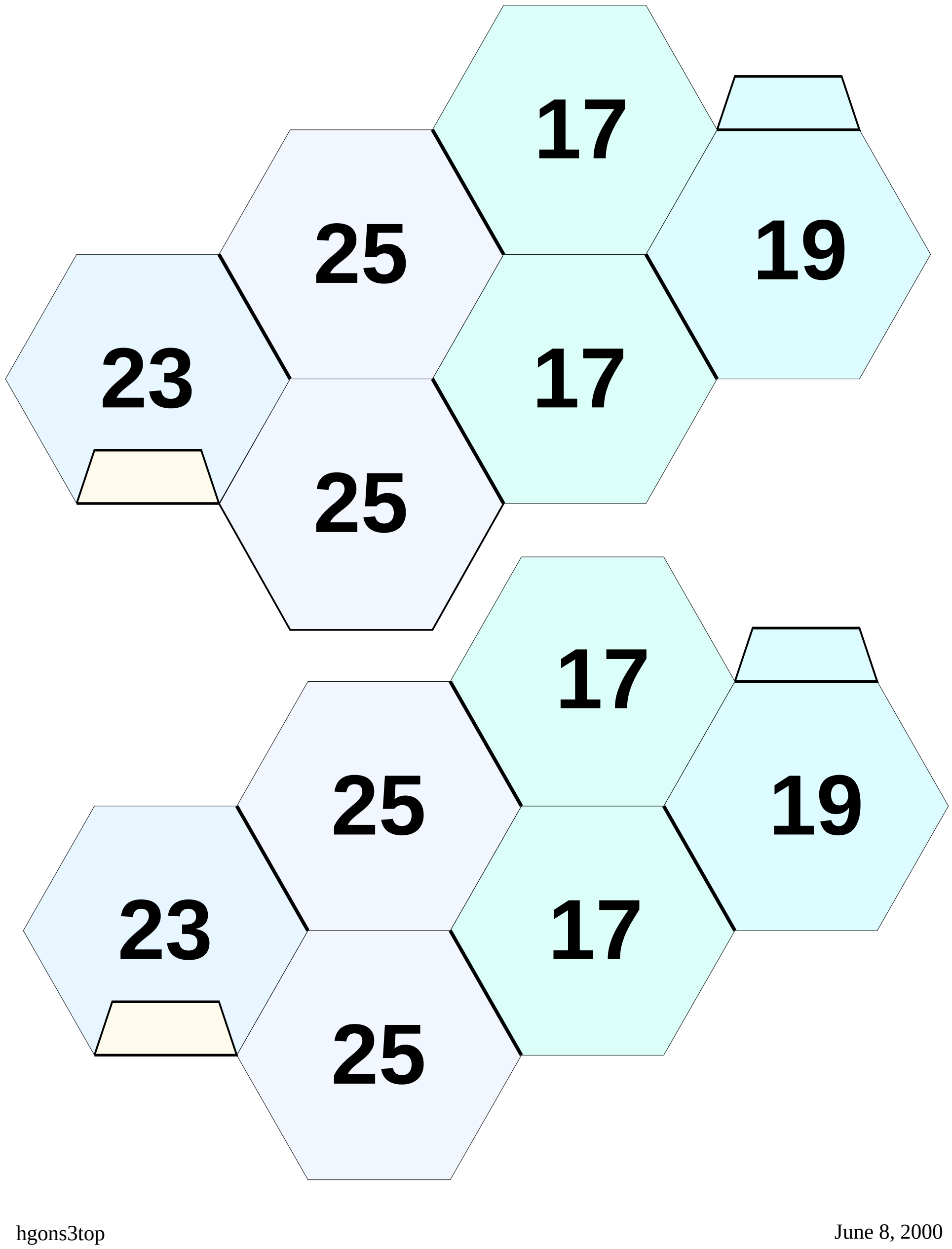



Figure 67: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level flessagon can be constructed. This is the third copy.

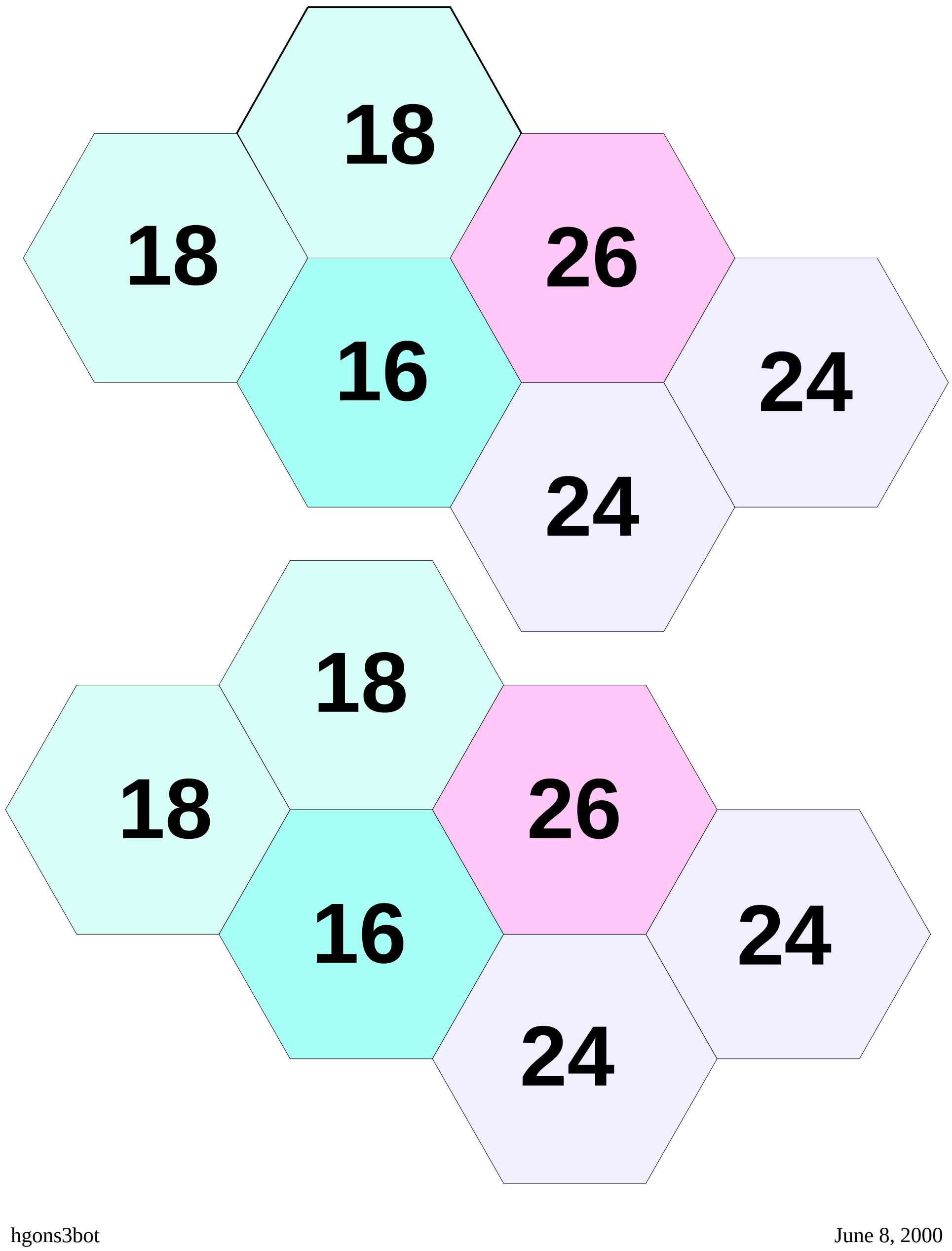



Figure 68: Bottom side of a second order flessagon.

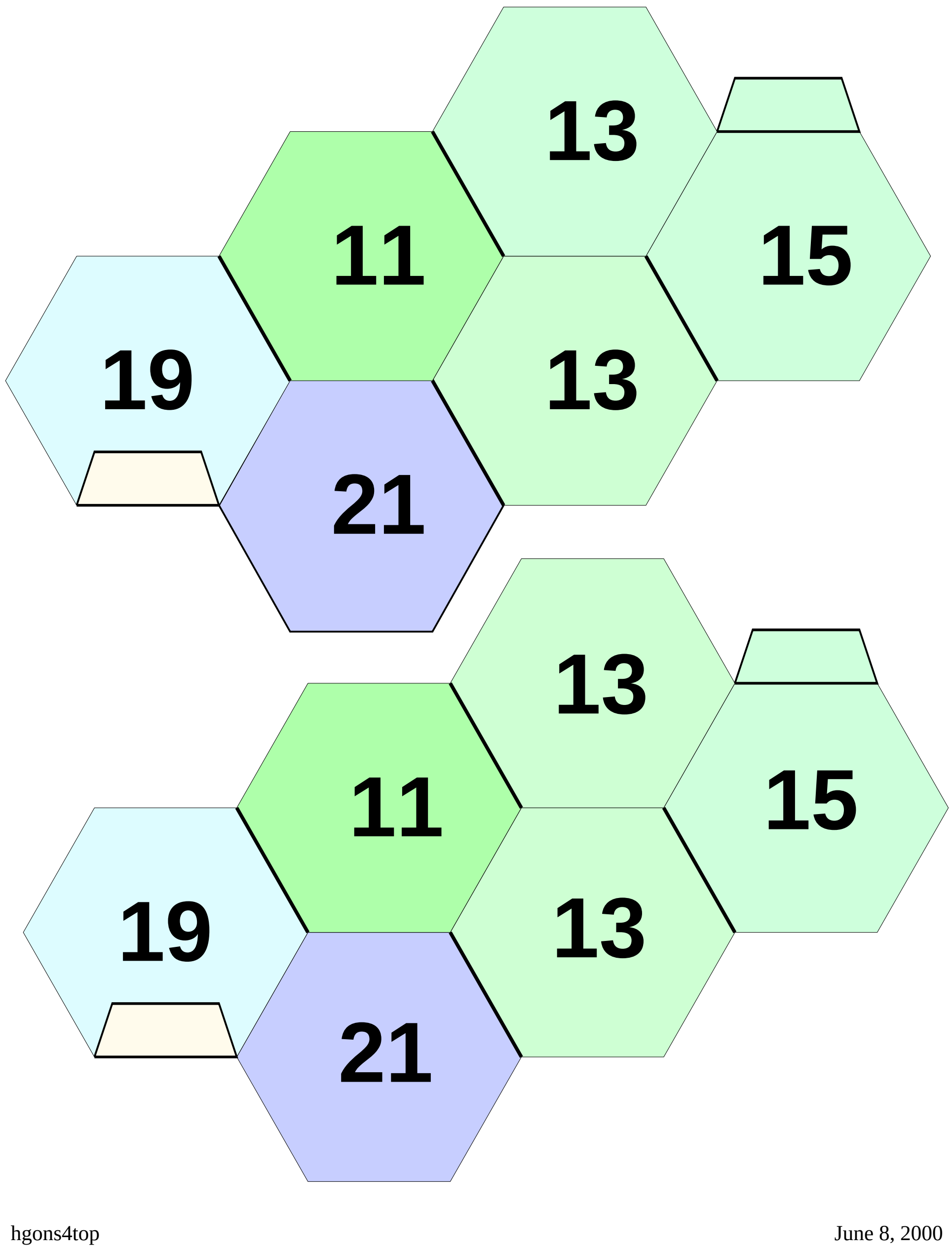



Figure 69: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level flessagon can be constructed. This is the fourth copy.

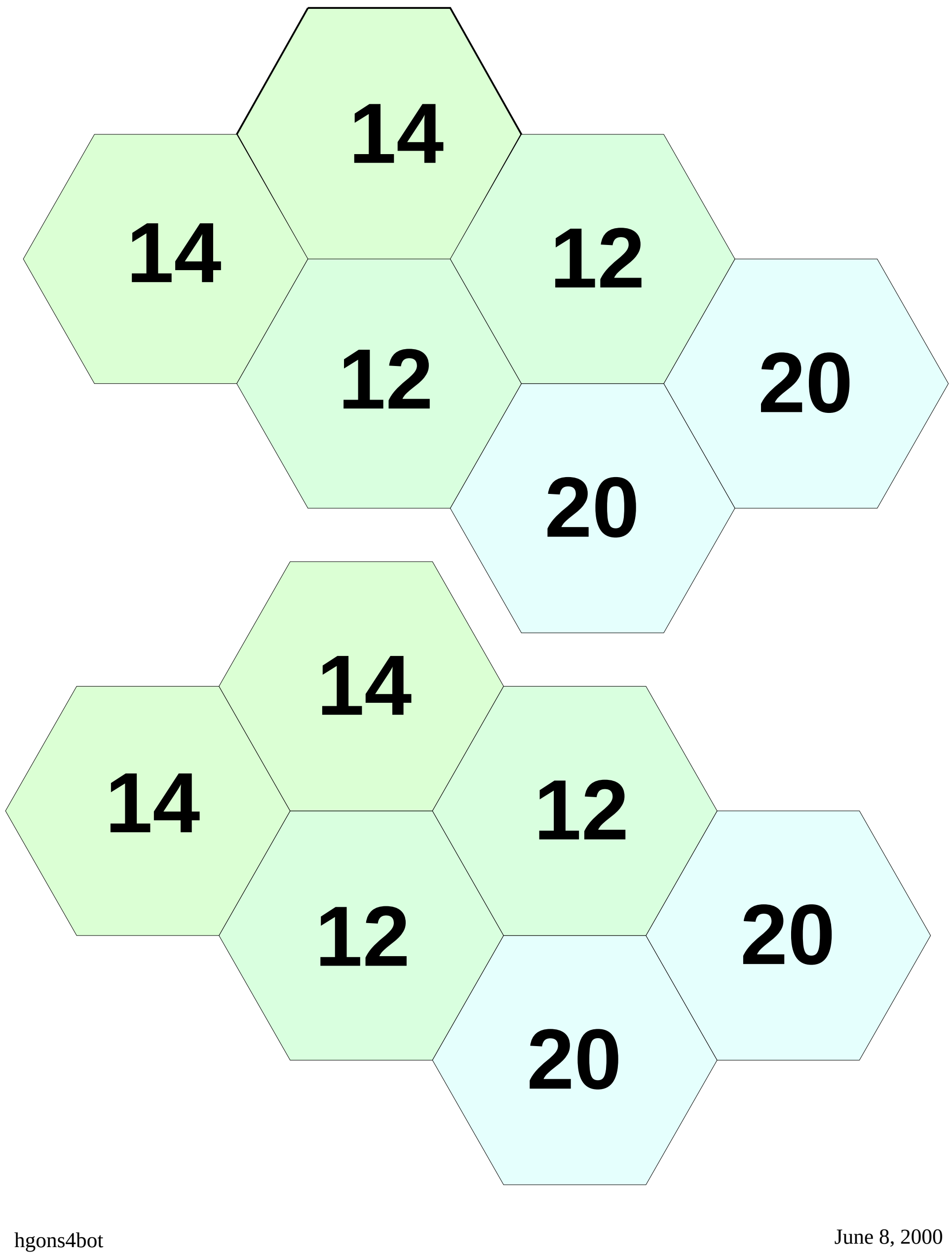



Figure 70: Bottom side of a second order flessagon.

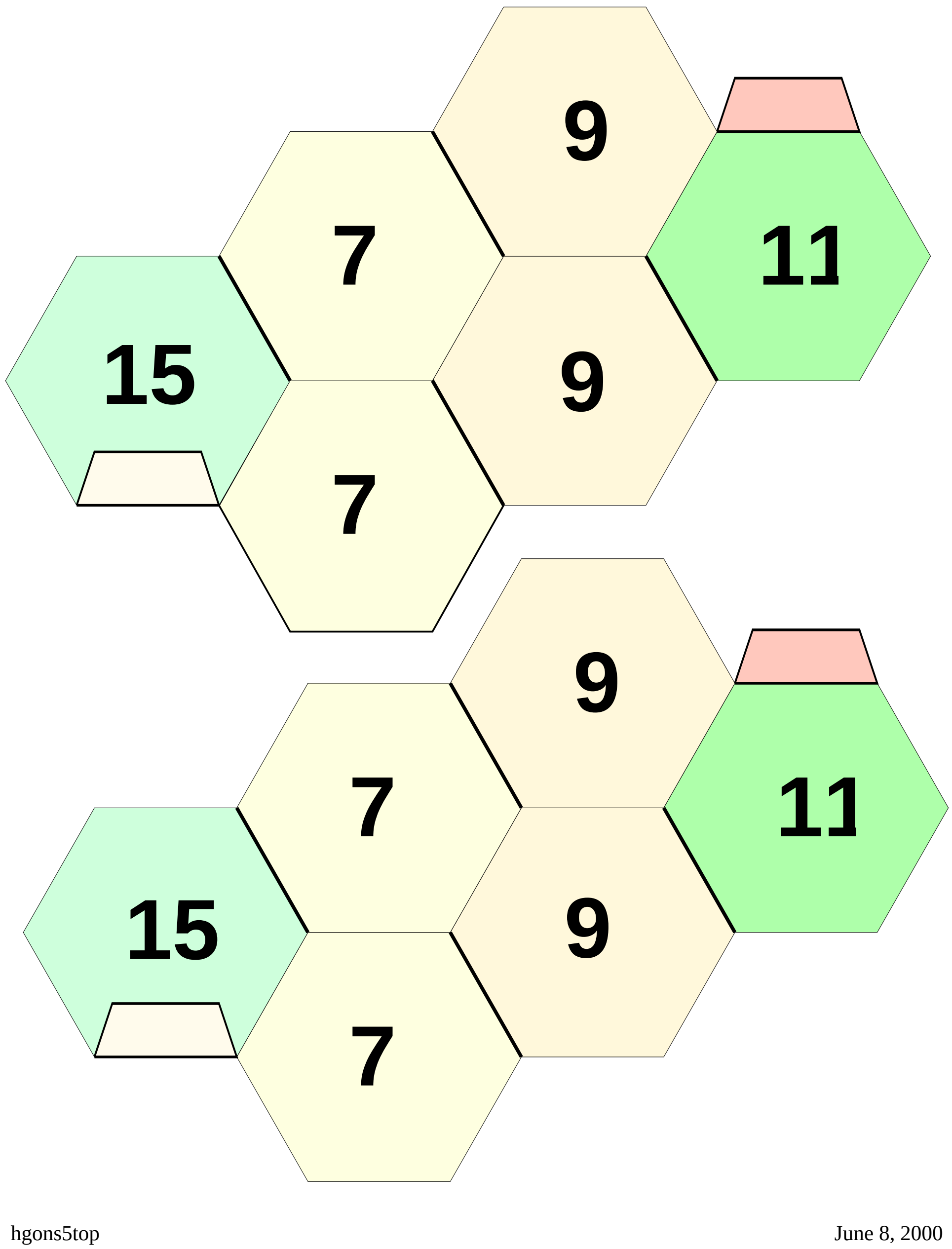



Figure 71: Top side of a segment which can be cut out, and by using five copies altogether, the strip for a second level flessagon can be constructed. This is the fifth copy.

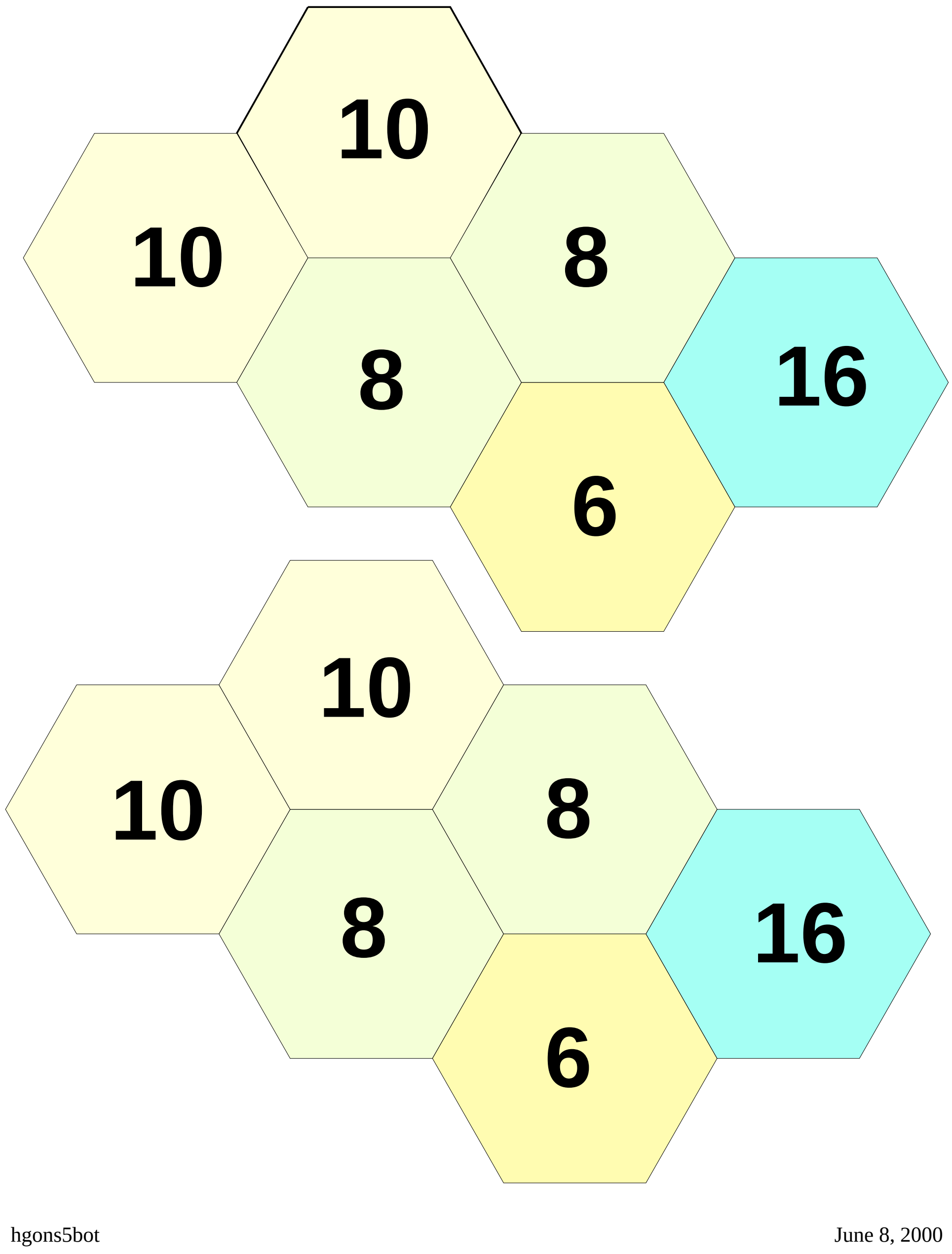



Figure 72: Bottom side of a second order flessagon.

# 13 First Level Heptagonal Flexagon

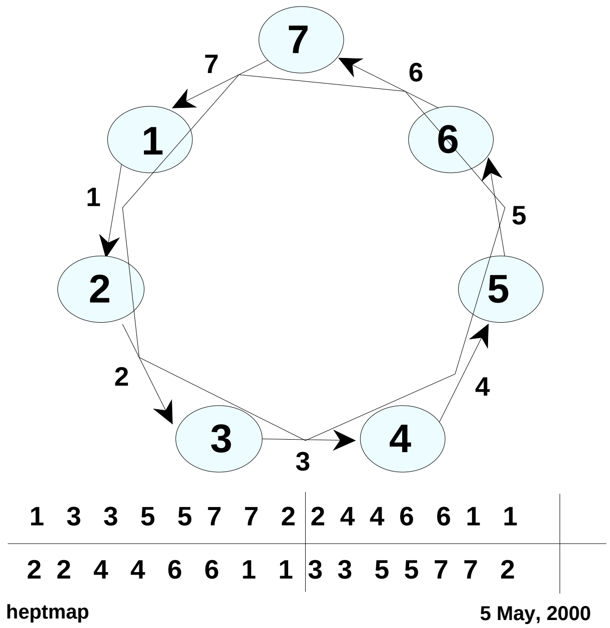



Figure 73: The first level heptagonal flexagon has 7 vertices.

The strip of polygons from which a flexagon is assembled embeds neatly into the plane for triangles through hexagons, although entire polygons will occasionally overlap when the rules of connection demand it. But the first and third of three consecutive heptagons will overlap slightly when successors joined by adjacent edges, which is the rule for normal flexagons.

If a strip is prepared in advance with the intention of folding it later on to get the flexagon, the overlapping heptagons can be trimmed slightly without spoiling the effect. Nevertheless in progressing onwards to octagons, nonagons, and so on, the overlap becomes increasingly severe, and it will probably be a good idea to prepare the polygons separately, or in sparser strips, joining them in later stages of the assembly.

Once folding the strip of polygons begins, the higher order polygons increasingly resemble circles, the hinges between adjacent polygons gradually rotating around the circumference. The result is a four bladed rosette formed from two sectors and four pats, in which the transferrence of subpats from one pat to another procedes with exceptional ease and clarity. However in compensation the flexagon operation of “rotation” requires increasingly more versatile gymnastics. More and more tubulations also become possible as the series progresses.

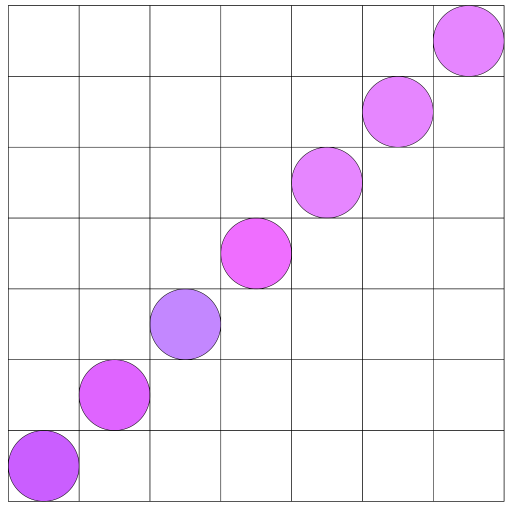


Figure 74: Permutation of the heptagons along the strip for a first level heptagonal flexagon.

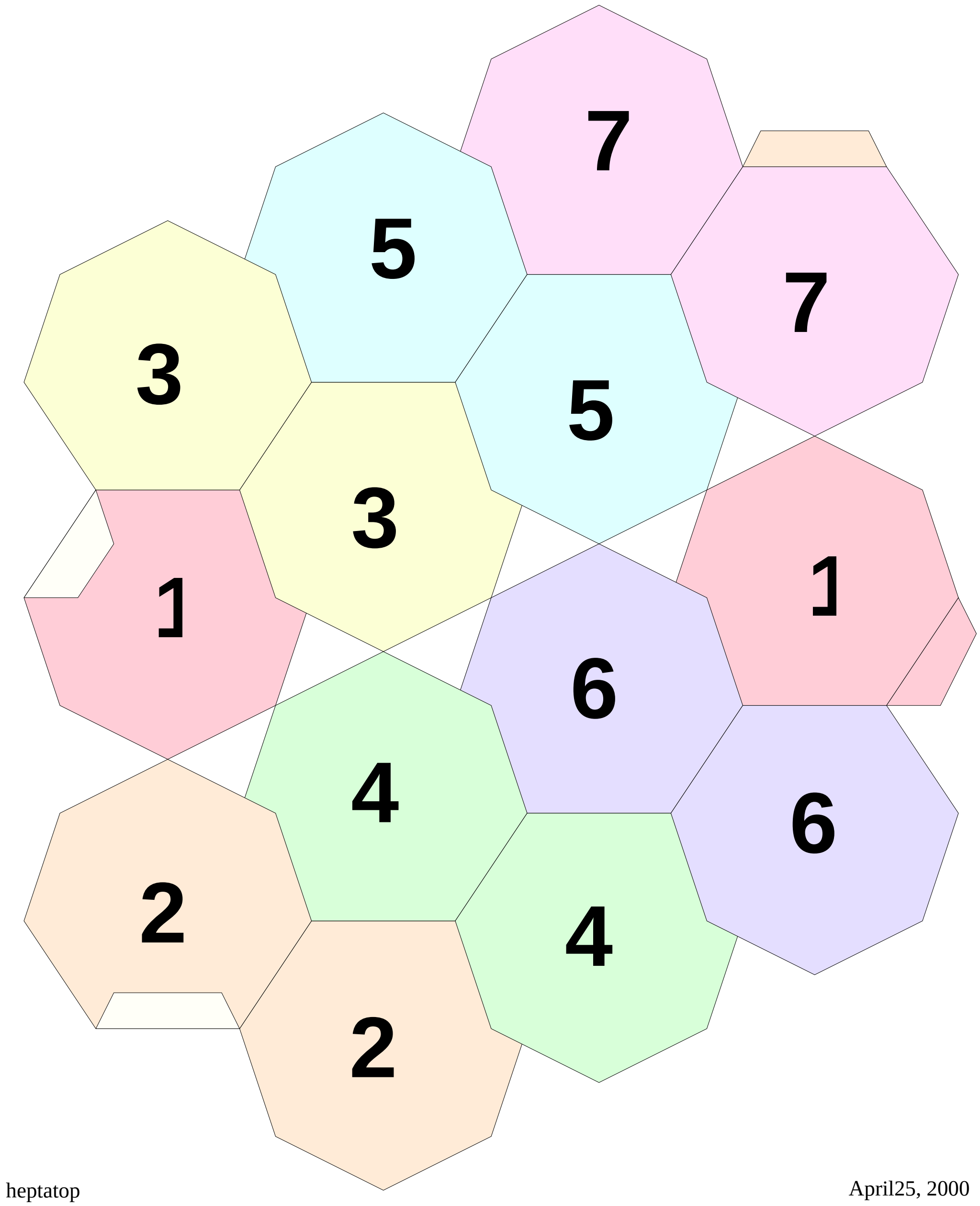



Figure 75: Top side of a heptagonal flexagon consisting of a single cycle.

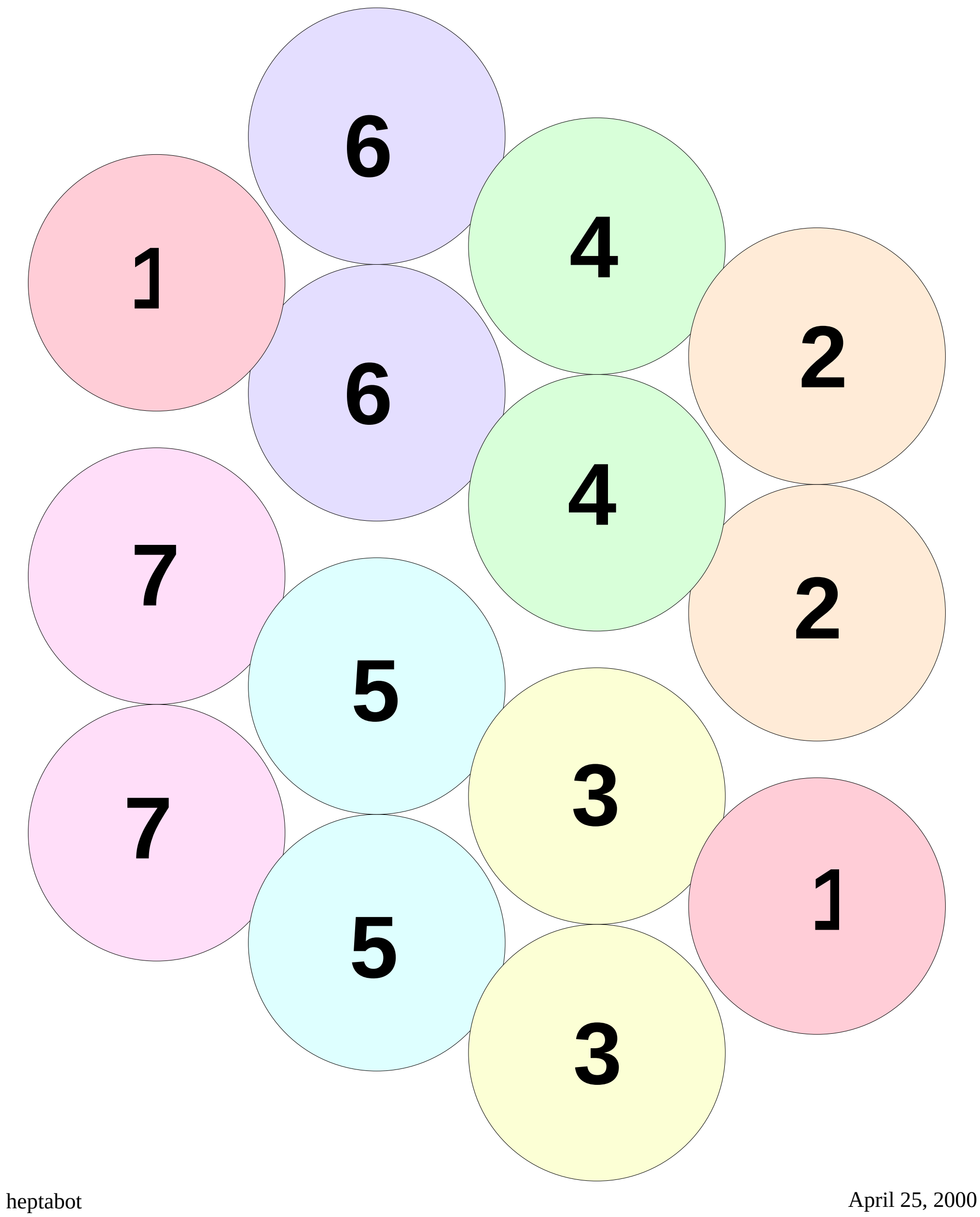



Figure 76: Bottom side of a heptagonal flexagon consisting of a single cycle.

# 14 First Level Octagonal Flexagon

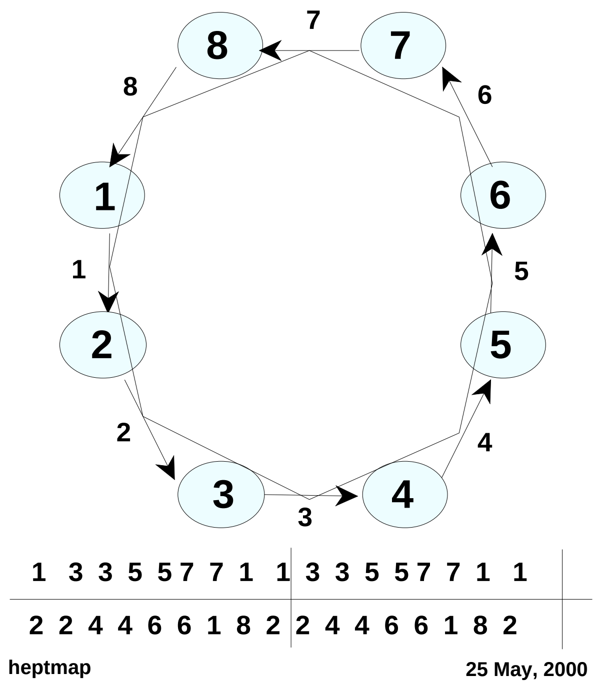



Figure 77: The first level octagonal flexagon has 8 vertices.

Although there is some overlapping of the individual octagons in a strip of octagons, it is still not severe, and is outweighed by the versatility of thye resulting octagonal flexagon. In any event, it is the relationship between successive hinges which matters in a flexagon, so the remainder of the polygon can be rearranged to suit convenience or aesthetics.

If it is considered important to preserve the full symmetrical polygons, they can be cut out individually and pasted toggether. Once the strip is folded, overlapping will no longer be a problem. If rapidity and efficiency of assembly is preferrd, corners can be cut but the flexagon will still work.

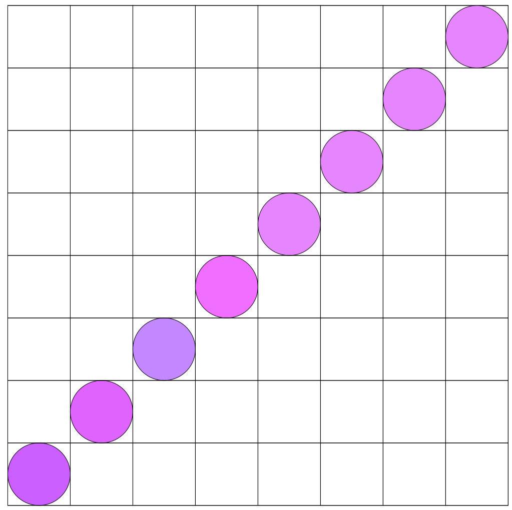


Figure 78: Permutation of the octagons along the strip for a first level octagonal flexagon.

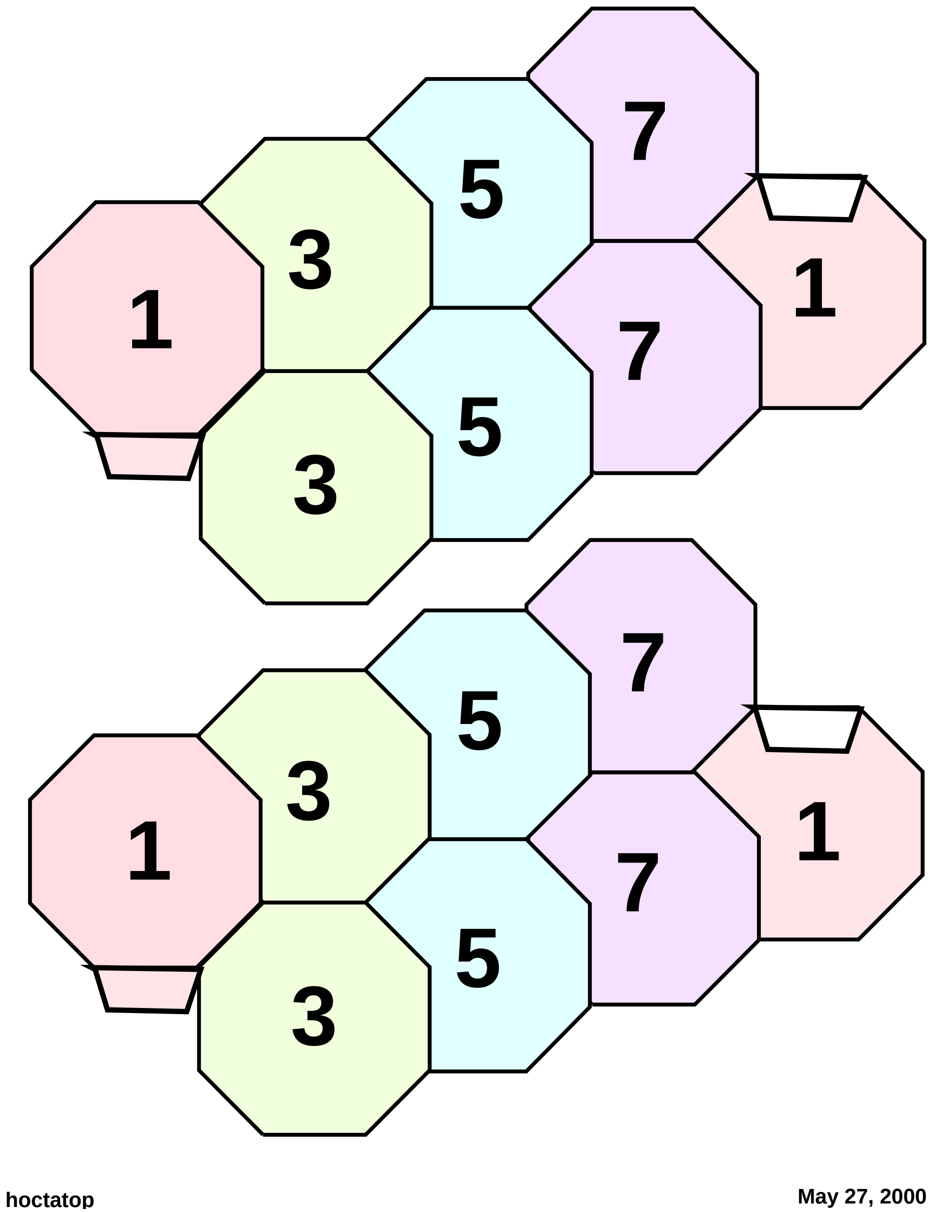



Figure 79: Top side of an octagonal flexagon consisting of one single cycle.

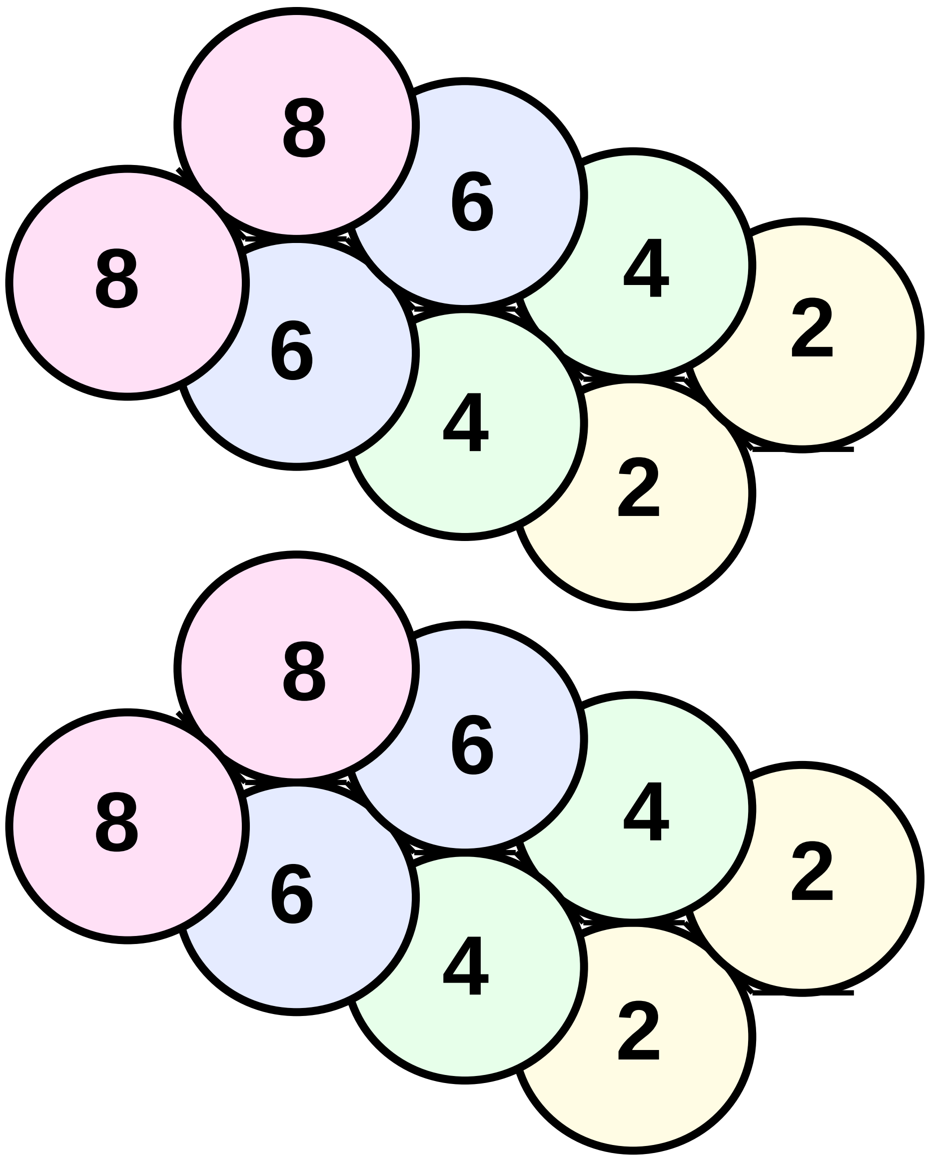



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Figure 80: Bottom side of an octagonal flexagon, consisting of one single cycle.

## 15 Mixed Flexagon

A pat is simply a wad of paper arising from any strip, straight or otherwise, which has been folded round and round; but in a consistent direction to keep it from springing apart when released. It is much more interesting for the pat to have a linear substructure, whereby it is perceived as a one-dimensional strip whose constitutents are similar wads. The simplest composite is a pair, and by taking a flexagon to be a pair of pats, the possibility exists to transfer one of the wads, or subpats, from one member of the pair to the other. In essence, that is the basis of Oakley and Wisner's definition of a flexagon.

Strictly, that is the definition of a triflexagon, because by letting one member of the overall pair be a triplet rather than a pair is the way to introduce tetraflexagons. Of course, to get a geometrically pleasing toy, all of this has to be realized via polygons fitting together around a common center giving a ring which may or may not want to lay flat.

Provided that the geometric constraints are met, there is no reason that the members of pairs cannot be triplets, or that the members of triplets cannot be pairs. That is particularly evident if one is willing to forego opening then up, and to regard a wad with internal structure as a monolithic block instead.

The simplest mixture has a map consisting of one triangle joined onto one square. The common meeting ground, if one were needed, would be a flexagon based on dodecagons, of which the simplest would be a ring of twelve.

### 15.1 Tukey triangles

### 15.2 Mixed permutations

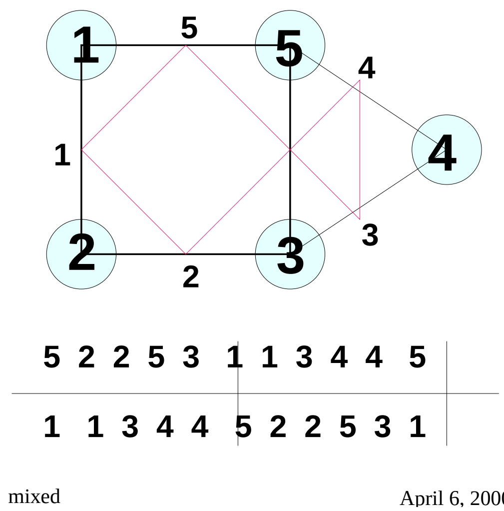


Figure 81: A four-cycle (tetraflexagon) and a three-cycle (triflexagon) combine to give a five faced mixed flexagon which is best represented in a dodecagon, the polygon which is their least common multiple.

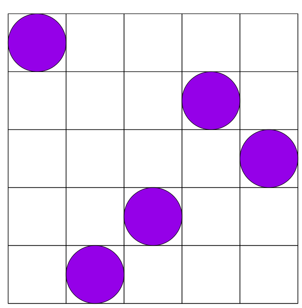

Figure 82: Permutation of the hexagons along the strip for a mixed flexagon.

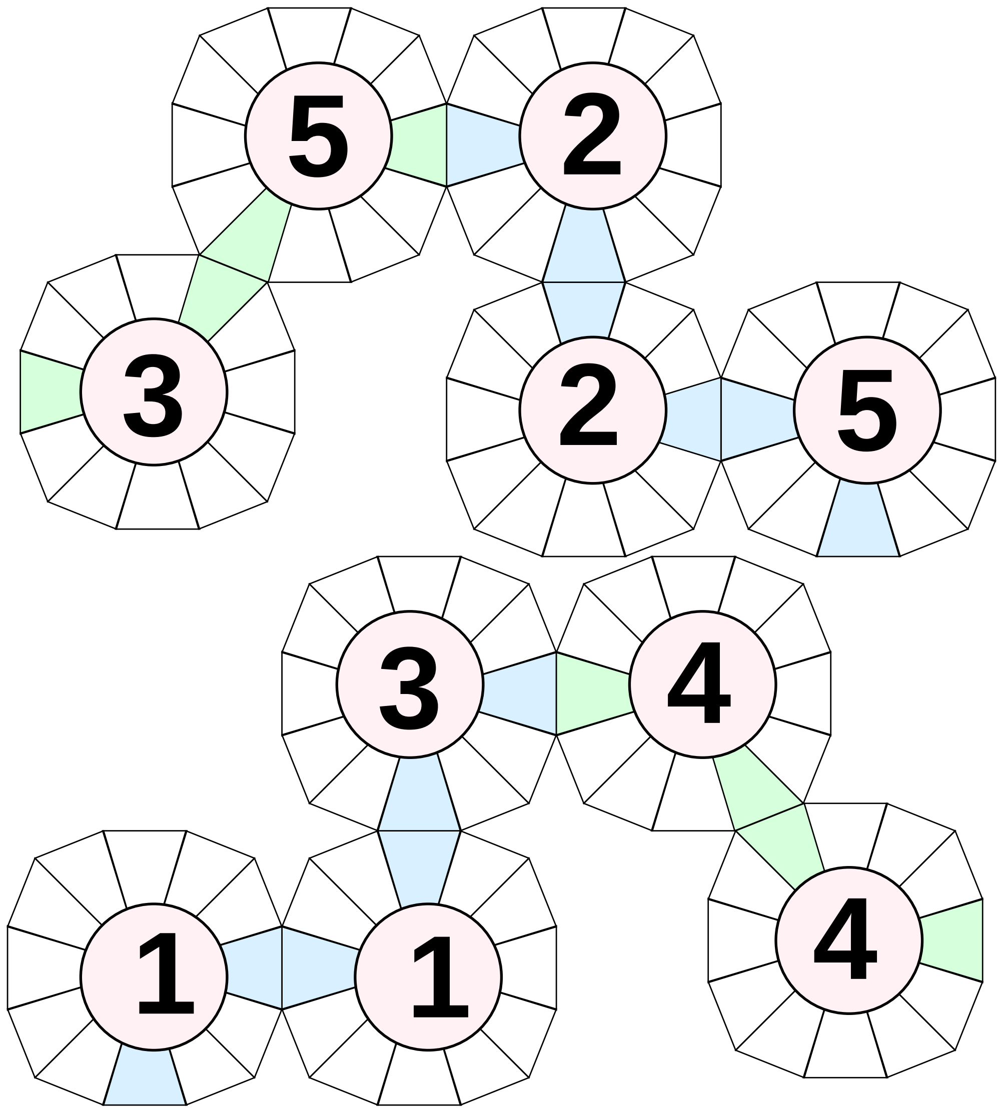


Figure 83: Top side of a dodecagon string from which the mixed flexagon may be assembled. Two sectors are shown, sufficient to make one flexagon. Since the flexagon is odd, the numbering moves to the opposite side from one sector to the other.

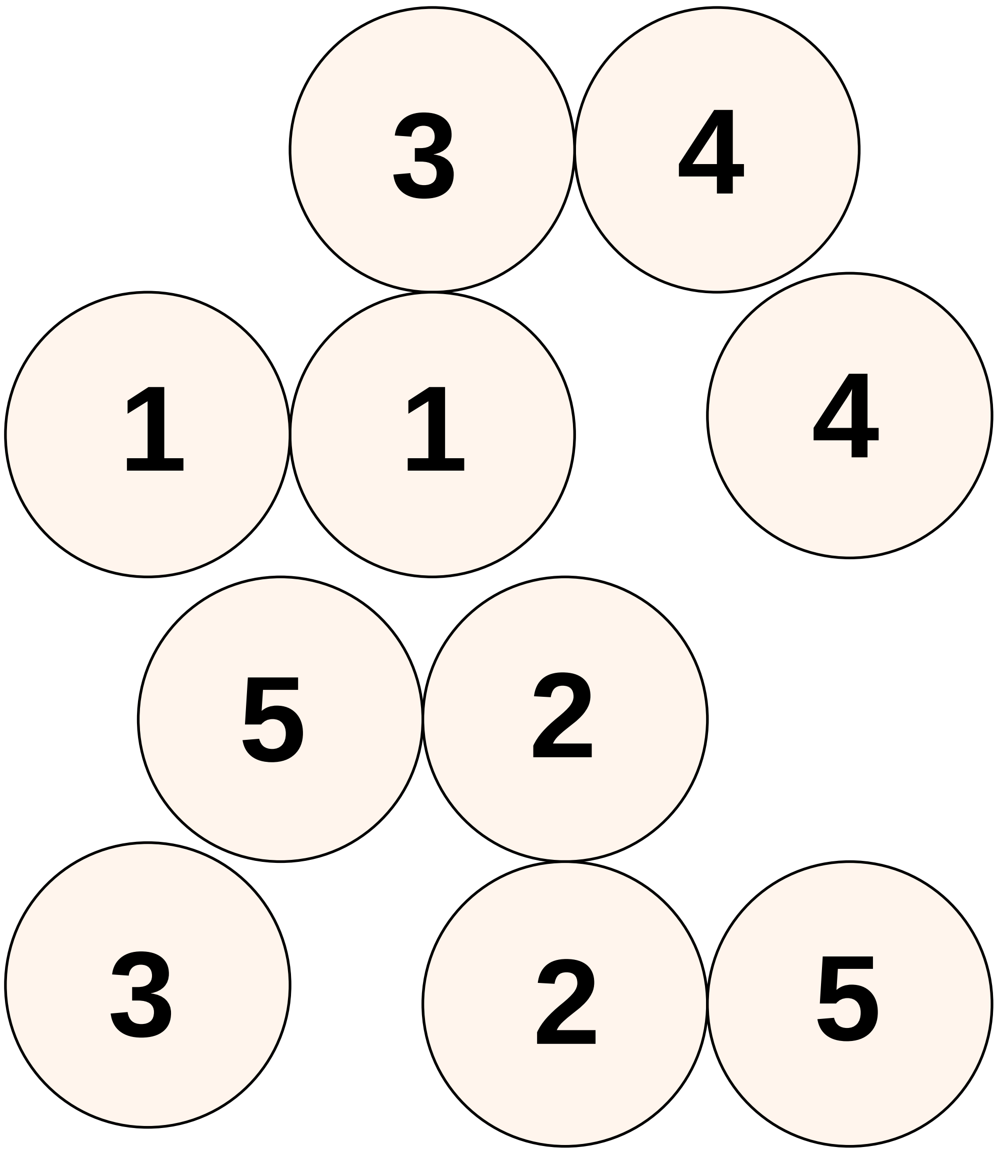


Figure 84: Bottom sides a dodecagon strip from which the mixed flexagon is assembled.

## 15.3 3-4-3 Tukey triangles

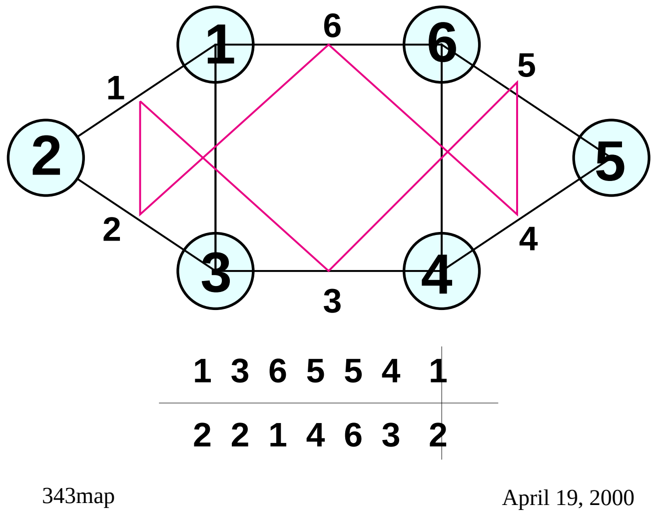


Figure 85: A three-four-three-cycle six faced mixed flexagon which also is best represented in a dodecagon, the polygon which is their least common multiple.

### 15.4 3-4-3 Mixed permutations

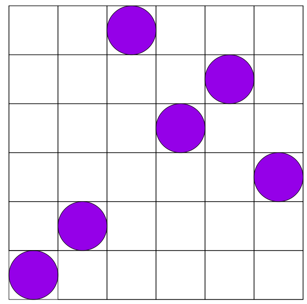

Figure 86: Permutation of the hexagons along the strip for a 3-4-3 mixed flexagon.

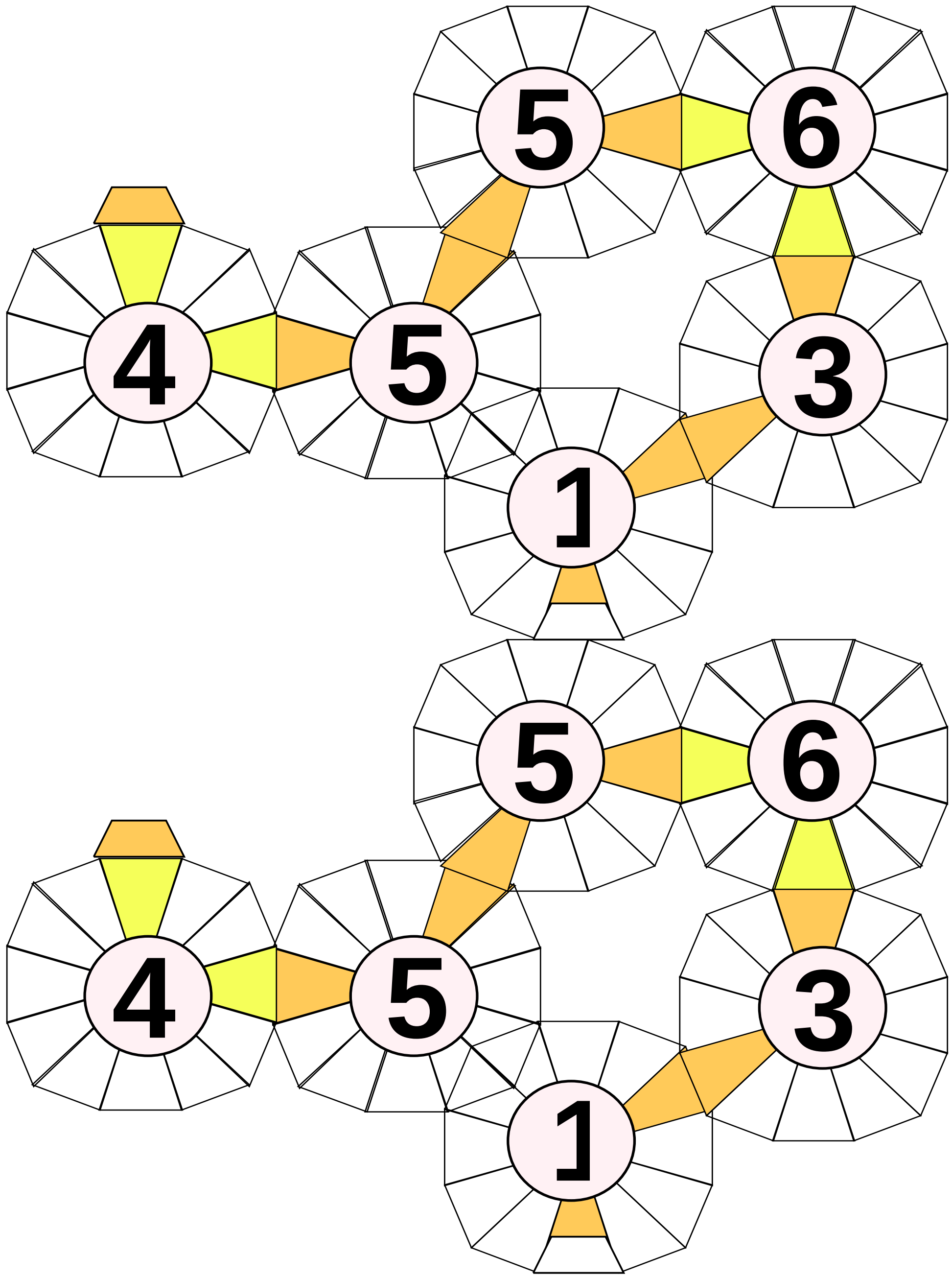


Figure 87: Top side of a dodecagon string from which the mixed flexagon may be assembled. Two sectors are shown, sufficient to make one flexagon. Since the flexagon is odd, the numbering moves to the opposite side from one sector to the other.

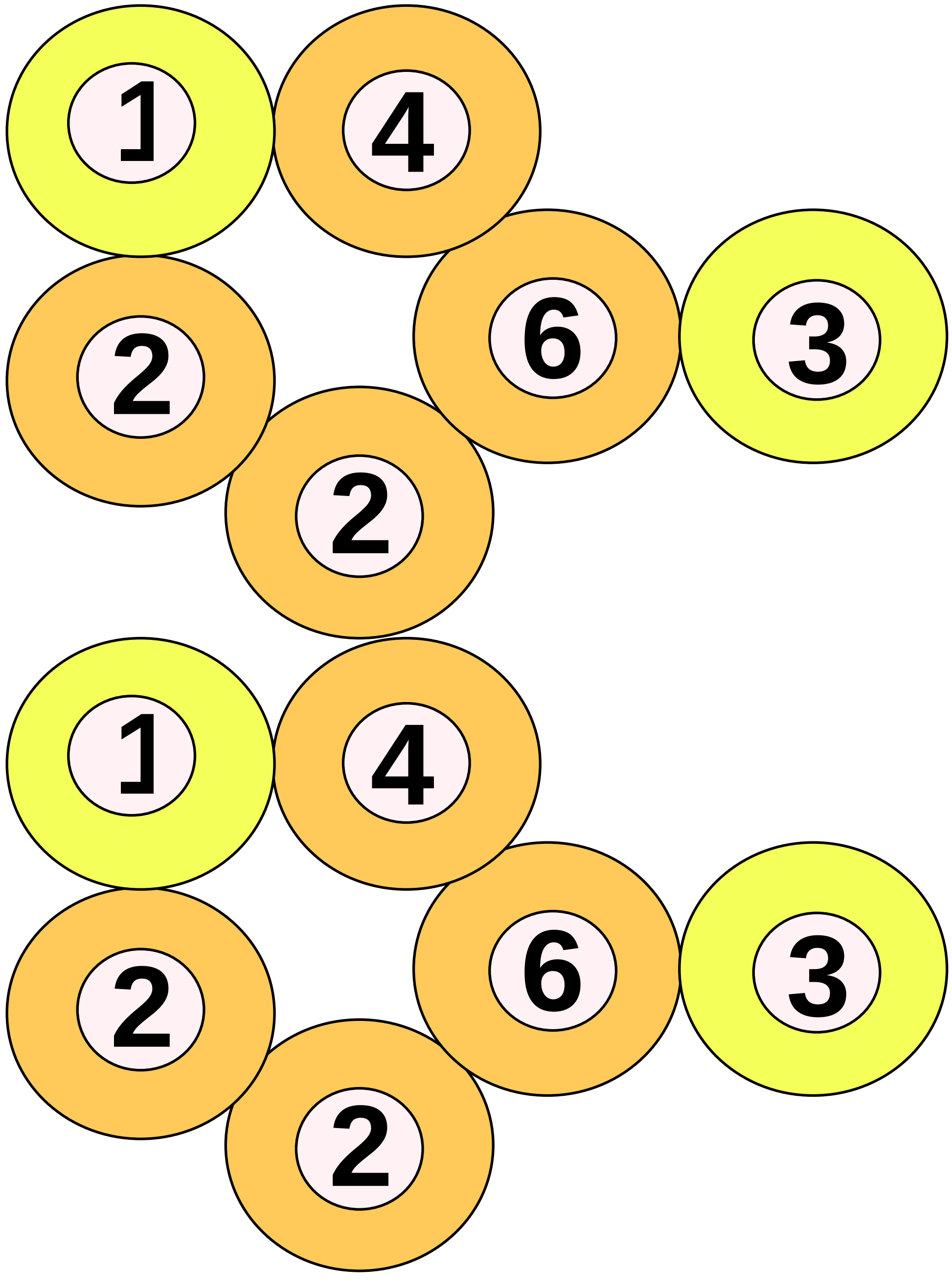


Figure 88: Bottom sides a dodecagon strip from which the mixed flexagon is assembled.